Physical modelling of wave propagation within a hollow cylindrical structure

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Research

School of Engineering

School of Engineering

- Advanced design and engineering
- **Sustainable infrastructure**
- Deakin Corrosion Research Centre (DCRC)
  - Prof Mike Tan

Strategic Centres & Institutes

- Institute for Intelligent Systems Research and Innovation
- Institute for Frontier Materials

Sustainable infrastructure

Structures

- Structural Rehabilitation
- **Damage identification**
- Structural Stability

Integrated Water

- Integrated water management
- Catchment management for sustainable ecosystem and communities
- Sustainable industrial and urban water management
- Resource recovery

Sustainable Energy

- Micro grid
- Renewable energy
- Energy neutral building

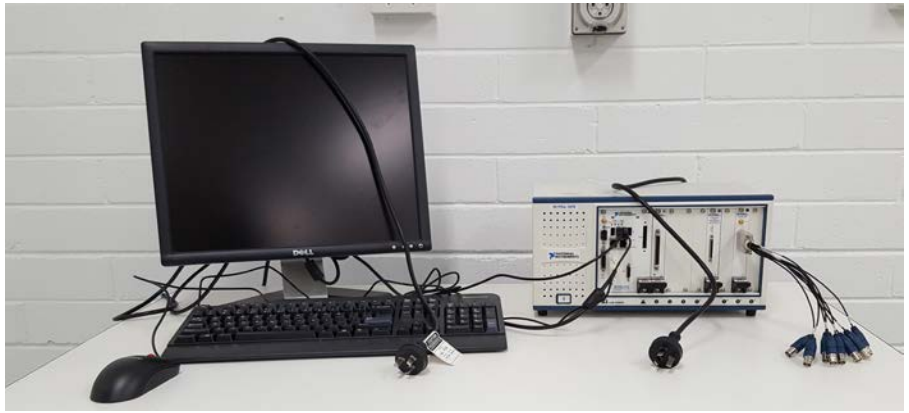
Sustainable infrastructure

Structures

- Structural Rehabilitation
  - Mike Tan
  - Riyadh Al-Ameri
  - Mahbube Subhani
- **Damage identification**
  - **Aman Muang Than**
  - **Timothy Bodisco**
  - **Mahbube Subhani**

Damage Identification

- Vibration based NDT
  - Steel
  - Timber
- Guided wave based NDT
  - Steel
  - Timber



Background

Physical modelling

Physical modelling of wave- Vibration based NDT

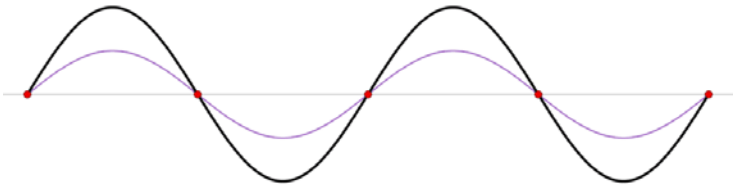
- Frequency of waves
  - Standing waves
  - Propagating wave
- Velocity of the wave
  - Standing wave
  - Propagating wave
- Attenuation of wave
  - Standing wave
  - Propagating wave
- Shape of input wave
  - Sinusoidal
  - Wavelet
- Vibration properties of the structure
  - Natural frequencies
- Reflection peak from boundary
  - Propagating wave

Physical modelling of wave- Vibration based NDT

Background

Standing and propagating wave

- Dominating frequency components
  - Standing wave
  - Propagating wave
- Velocities of induced frequency components
  - Standing wave
  - Propagating wave
- Attenuation of induced frequency components
  - Standing wave
  - Propagating wave



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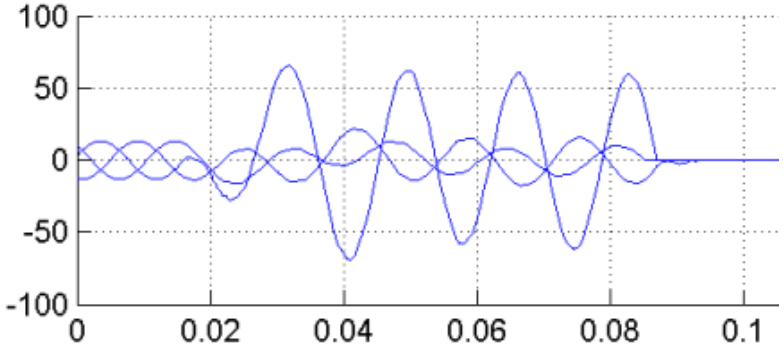
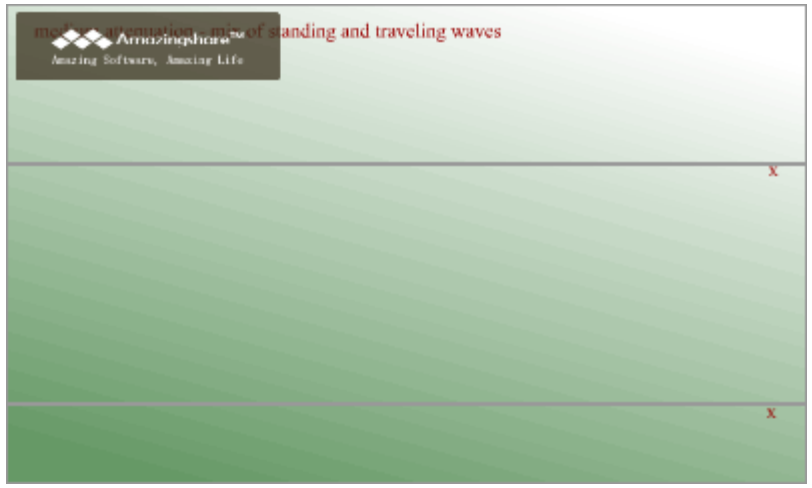


http://mathworks.com



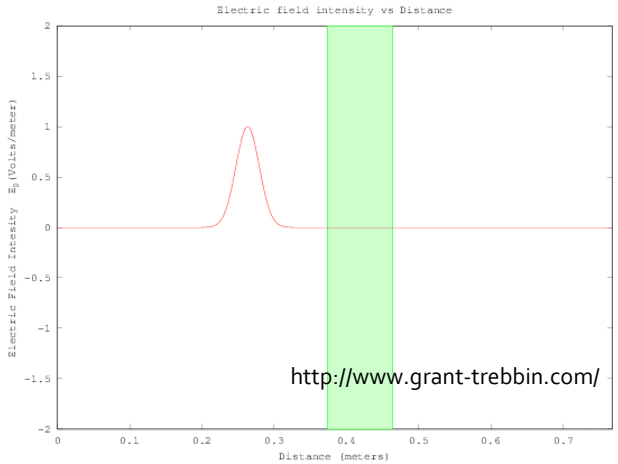
http://mars.gmu.edu/

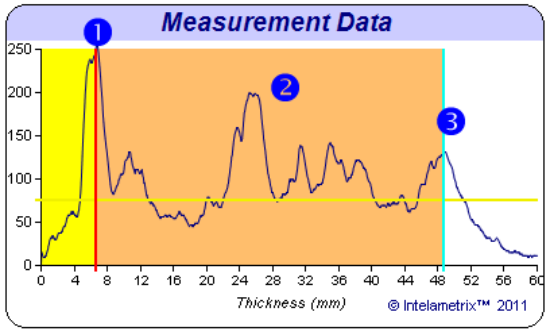
Background

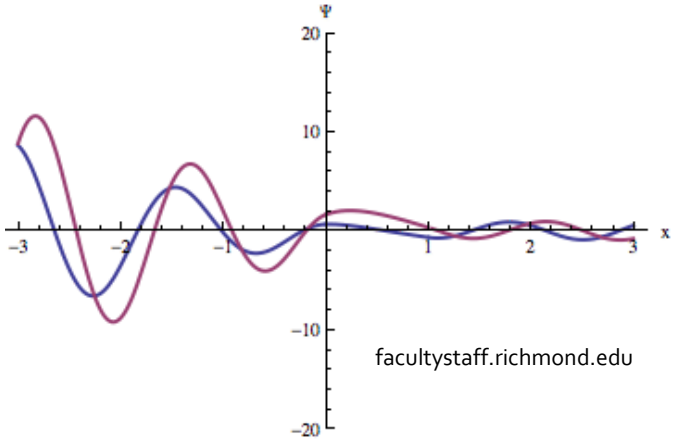
Reflection peak

Physical modelling of wave- Vibration based NDT

- Shape of input wave
  - Single sinusoidal
  - Wavelet
- Vibration properties of the structure
  - Natural frequencies
- Reflection peak from boundary
  - Propagating wave







Current modelling techniques

Advantages

Finite Element Method (FEM)

- Accurate
- Visual (time domain)
- Availability of software

Spectral FEM

- Computationally fast
- Physics of wave propagation can be understood
- Both frequency and time domain data can be obtained easily

Current modelling techniques

Limitation

Finite Element Method (FEM)

- High density meshing required
- Exact materials properties should be known
- Difficult to understand physics of the wave (primarily time domain based)
- Computationally expensive

Spectral FEM

- Selection of appropriate theory is vital
- One dimensional simplification (2 or 3D equations are complex)
- Across the cross-section data cannot be obtained
- Primarily developed for solid cylinder
- For hollow cylinder, physics of longitudinal wave propagation is available

Physical modelling of wave- Vibration based NDT

Present Study

Simplified Physical Modelling

Benefits

- Physics of wave propagation
  - Frequency of standing and propagation wave
  - Velocity of the standing and propagation wave
  - Attenuation of waves
  - Natural frequencies of the structures
  - Reflection peaks of propagating wave
- Based on experimental signal
  - Exact material properties are not required

Physical Modelling

Scope

- Current scope
  - Hollow cylinder is only considered
  - Isotropic material (steel)
  - Single frequency sinusoidal input is assumed from hammer impact
  - Computationally expensive (more iteration for convergence)
  - Traction free boundary conditions
- Future scope
  - Input signal will be arbitrary
  - Iteration loop will be optimised
  - Extended to anisotropic material (wood)
  - Embedded in solid and pipe flowing full/partially

Experimental test

- Uni-axial accelerometer: 3
- Transverse impact (bending wave)
- Sampling time: 50 ms (micro second)
- Signal length: 1 s

- Boundary:
  - Simply supported
  - Empty

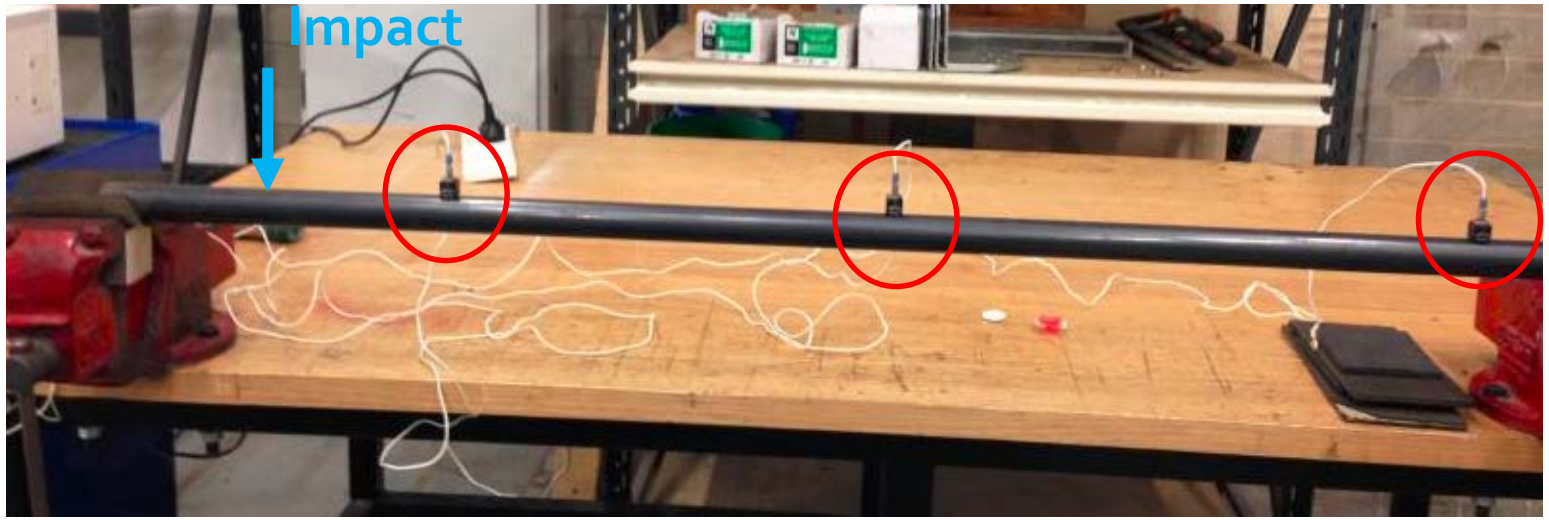


Fig: Healthy pipe with accelerometers (**30, 70, 120 cm from the left support**)

Experimental test

- Steel pipe
- Length: 1.3 m
- Outside dia: 34 mm
- Thickness: 3 mm

- Damage pipe: Corroded
- Weight:
  - Healthy: 3.033 kg
  - Corroded: 2.785 kg

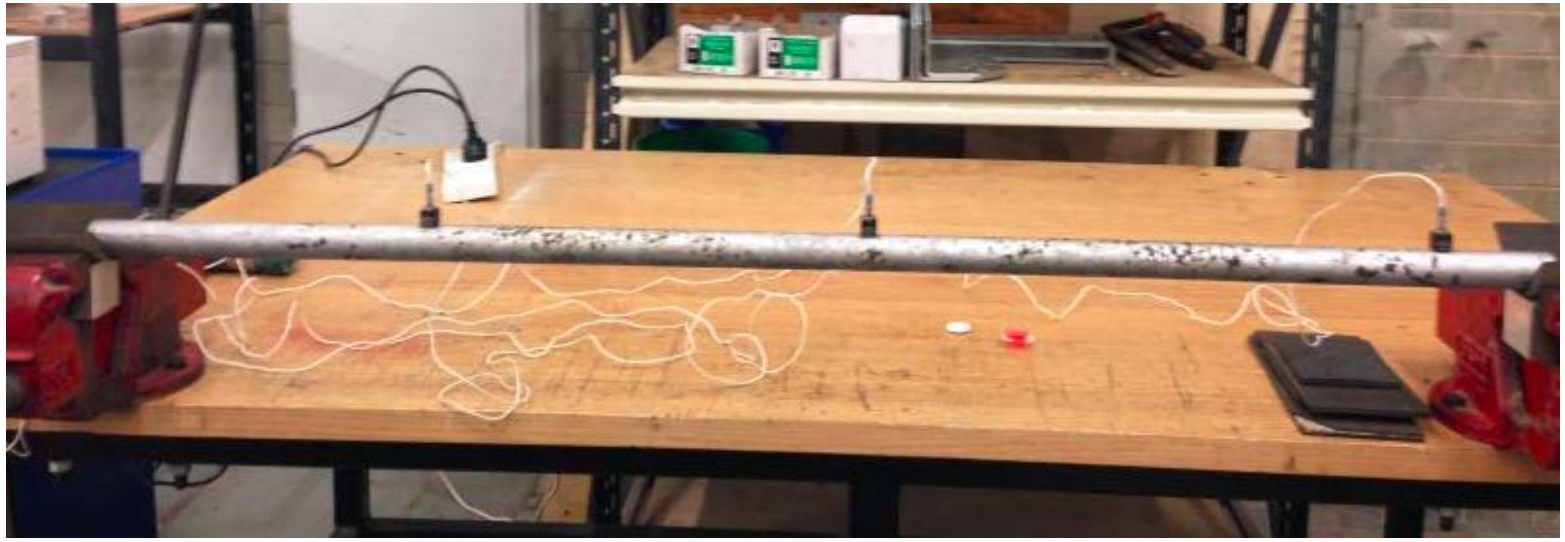
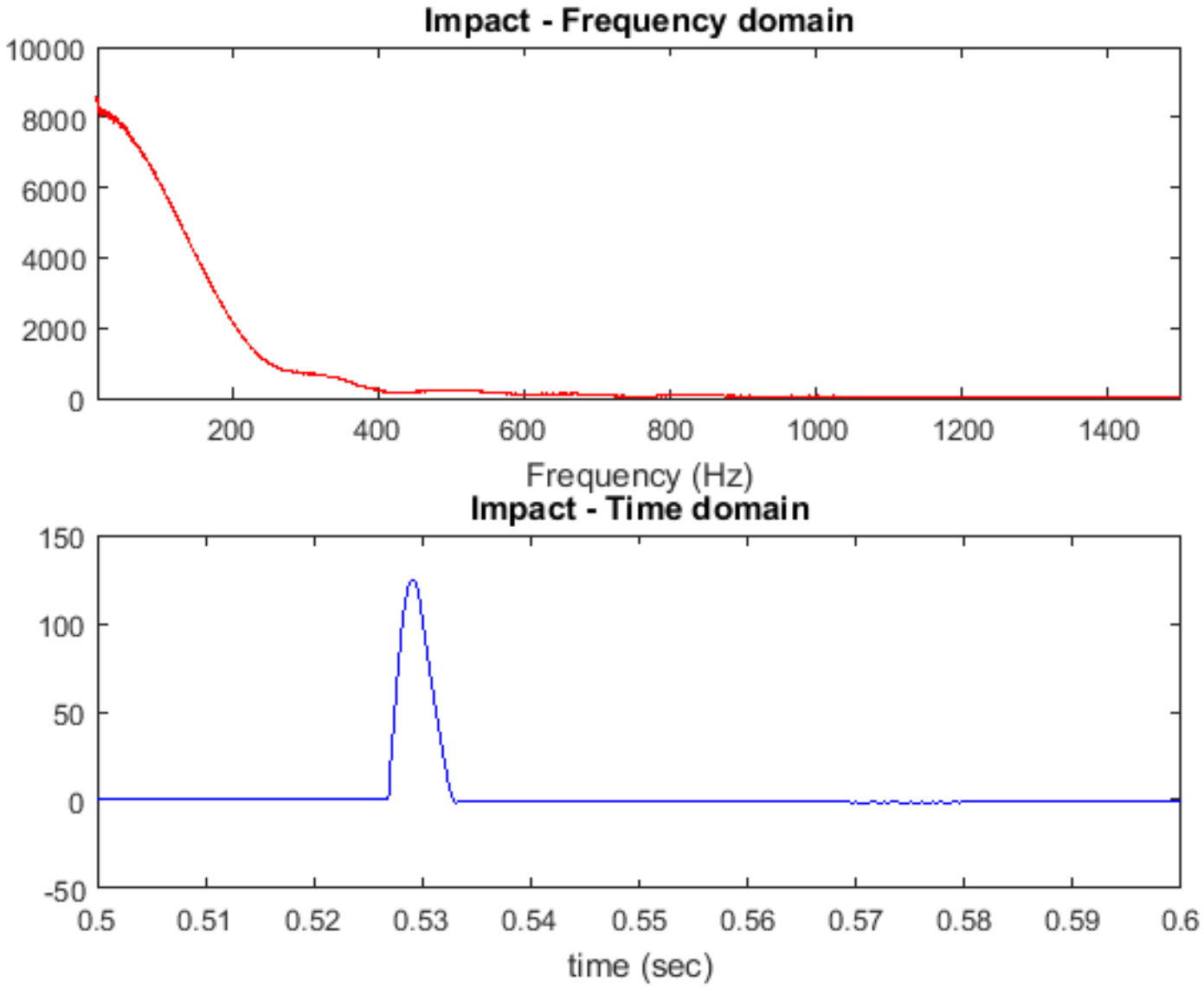


Fig: Damage pipe with accelerometers (30, 70, 120 cm from the left support)

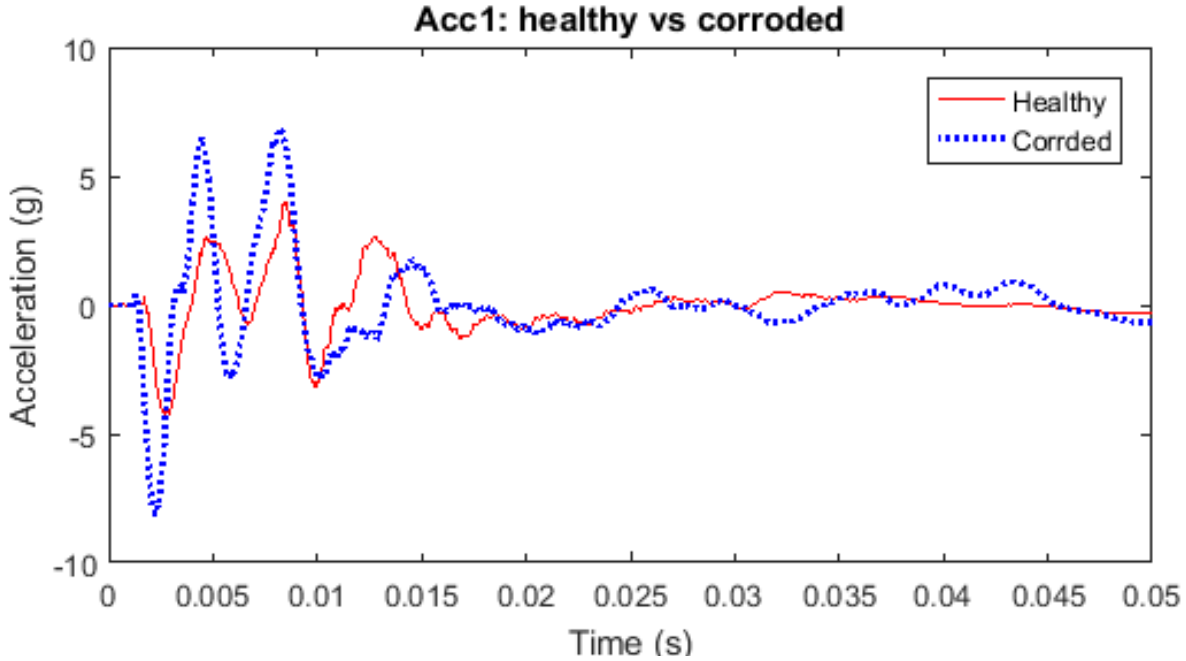
Experimental test

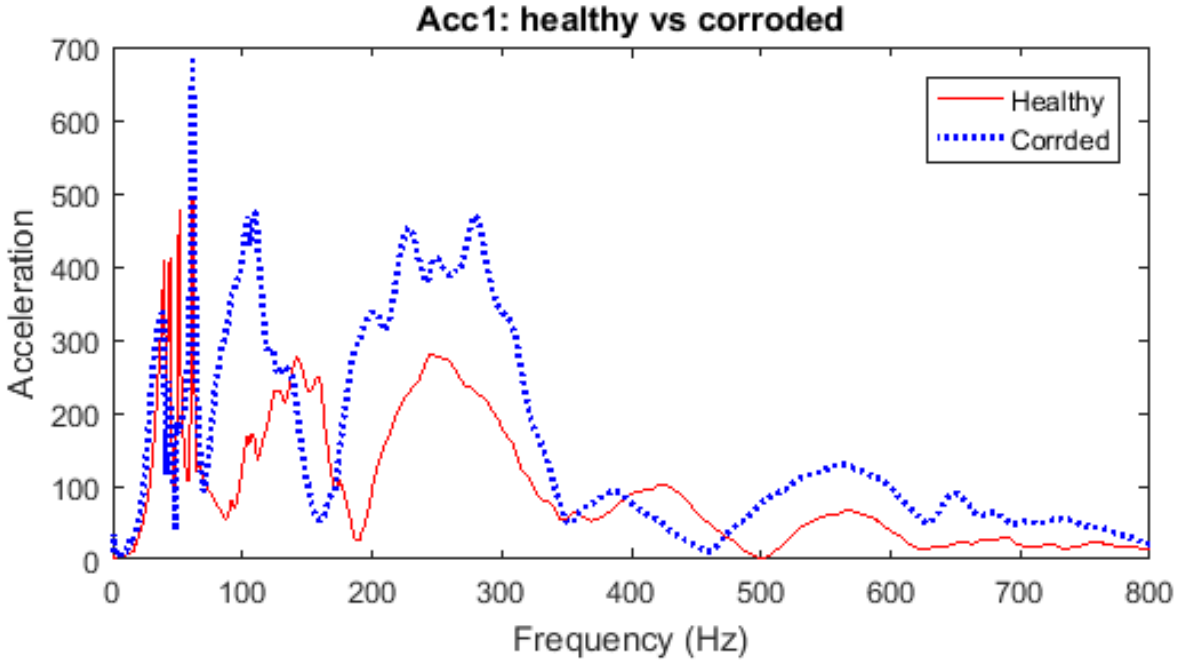
Impact hammer



Experimental test

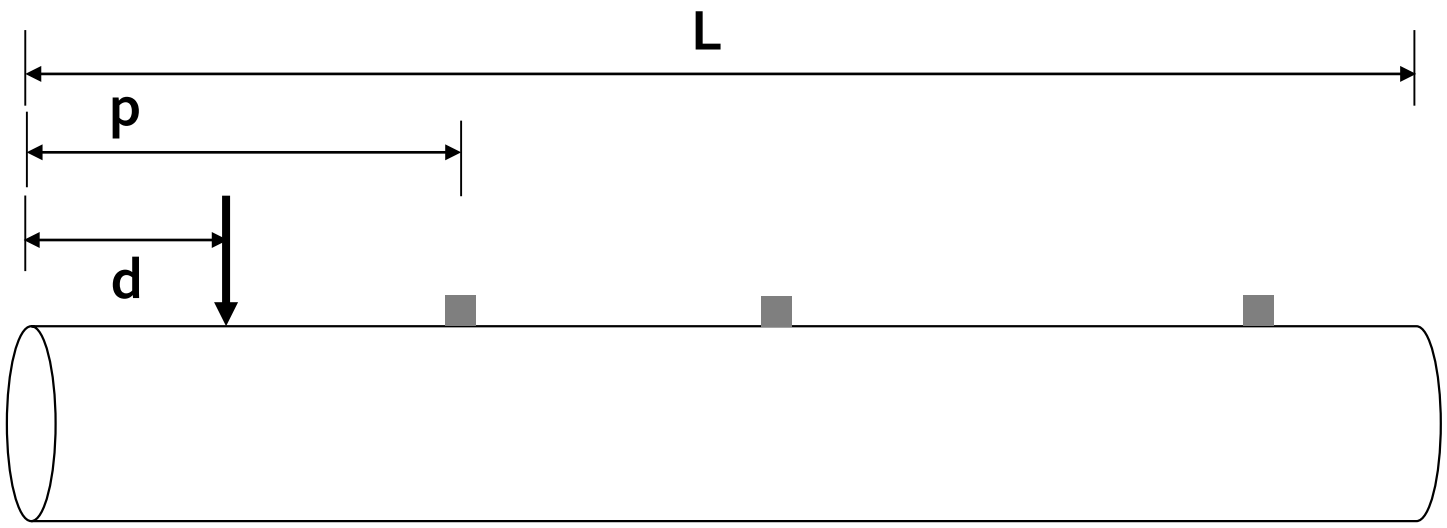
Accelerometers





Modelling

Parameters Notation



- ω = frequency
- v = velocity
- k = decay

- 1 = related to propagating wave
- 2 = related to standing wave
- 3 = related to natural frequency

Generation of the Physical Model

Model

Propagating Wave

- Originates at the point of impact
- Travels in along the pipe in both directions
- Reflects from the boundaries and returns
- Travelling wavelet visible to the accelerometers only when the wavelet if travelling past them

$$s_1(t) = \begin{cases} Ae^{-k(t-\delta)}\sin(2\pi\omega(t-\lambda_r)+\phi) & \lambda_r \leq t \leq \lambda_r + \frac{1}{\omega} \\ 0 & \text{Otherwise} \end{cases}$$

$$\lambda_0 = \delta + \frac{p-d}{v_1} \qquad \lambda_r = \begin{cases} \lambda_{r-1} + \frac{2(L-p)}{v_1} & \text{for } n = 1 \\ \lambda_{r-1} + \frac{2p}{v_1} & \text{for } n = 2 \end{cases}$$

$$A = A \cdot \alpha_n$$

$$\phi = \phi + P_n$$

Generation of the Physical Model

Model

Standing Wave

- Assumptions:
  - propagating wave causes standing waves to occur at the boundaries of the pipe
  - have a different velocity, frequency and decay rate, compared to the propagating wave
  - have the same amplitude and phase as the propagating wave that generated it

$$st_i(t) = \begin{cases} 0 & t < \delta_i - \frac{d_n}{v_2} \\ A_i e^{-k_2(t-\delta_i)} \sin(2\pi\omega_2(t - \delta_i - \frac{d_n}{v_2}) + \phi_i) & t \geq \delta_i - \frac{d_n}{v_2} \end{cases}$$

Natural frequency

- Low frequency caused by the impact of the hammer

$$f(t) = \begin{cases} 0 & t < \delta + \frac{p-d}{v_1} \\ A_3 e^{-k_3(t-\delta)} \sin(2\pi\omega_3(t - \delta) + \phi_3) & t \geq \delta - \frac{p-d}{v_1} \end{cases}$$

Generation of the Physical Model

Model

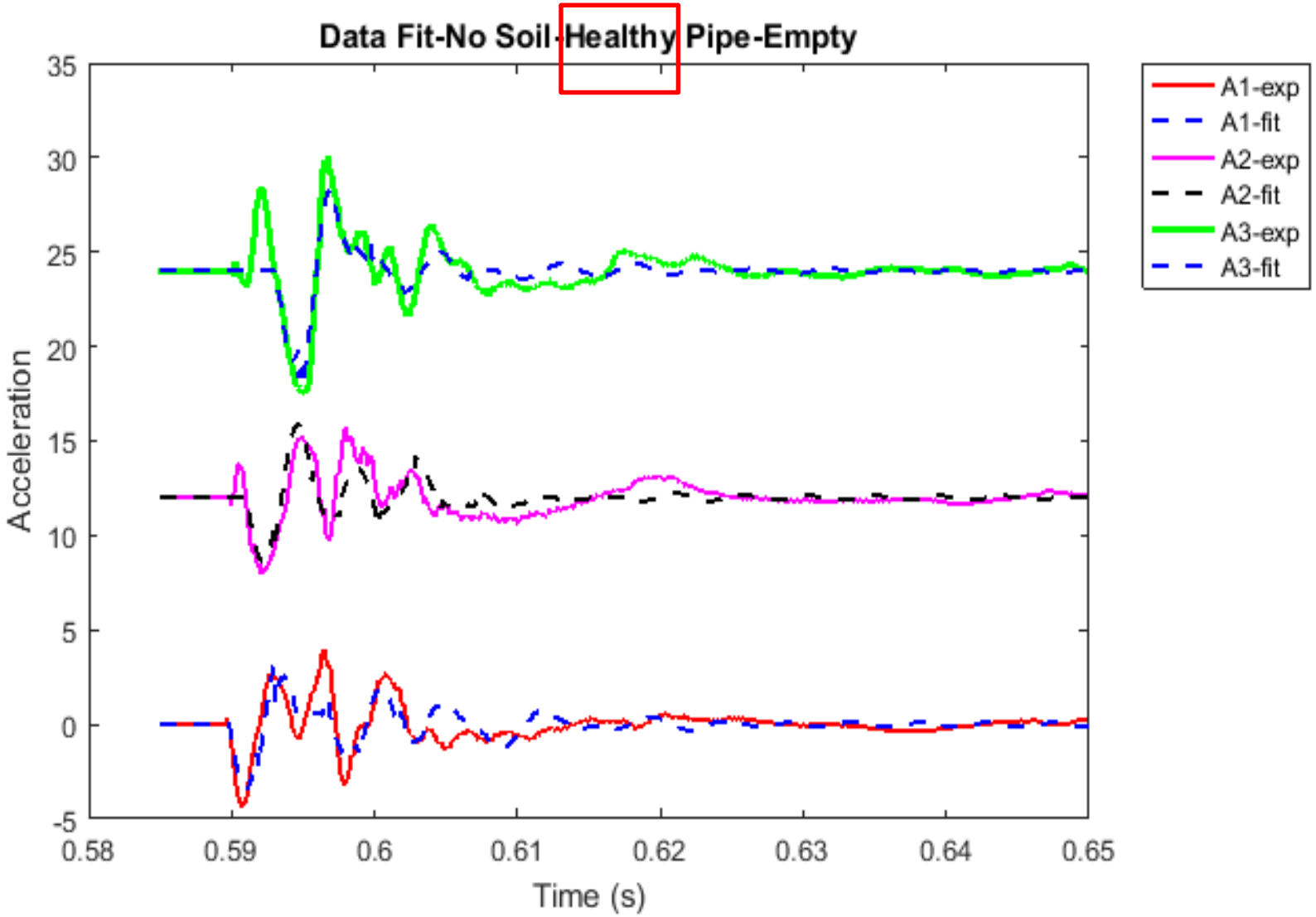
Building the model

- Building the signal
  - the summation of all the components, in each direction the signal is propagating
- Determination of the most likely parameters
  - the approach has been to optimise the fit of the model to the signal using a Genetic Algorithm
  - the cost function is a square difference, therefore this method has strong parallels to a least squares solution

Results

Healthy pipe

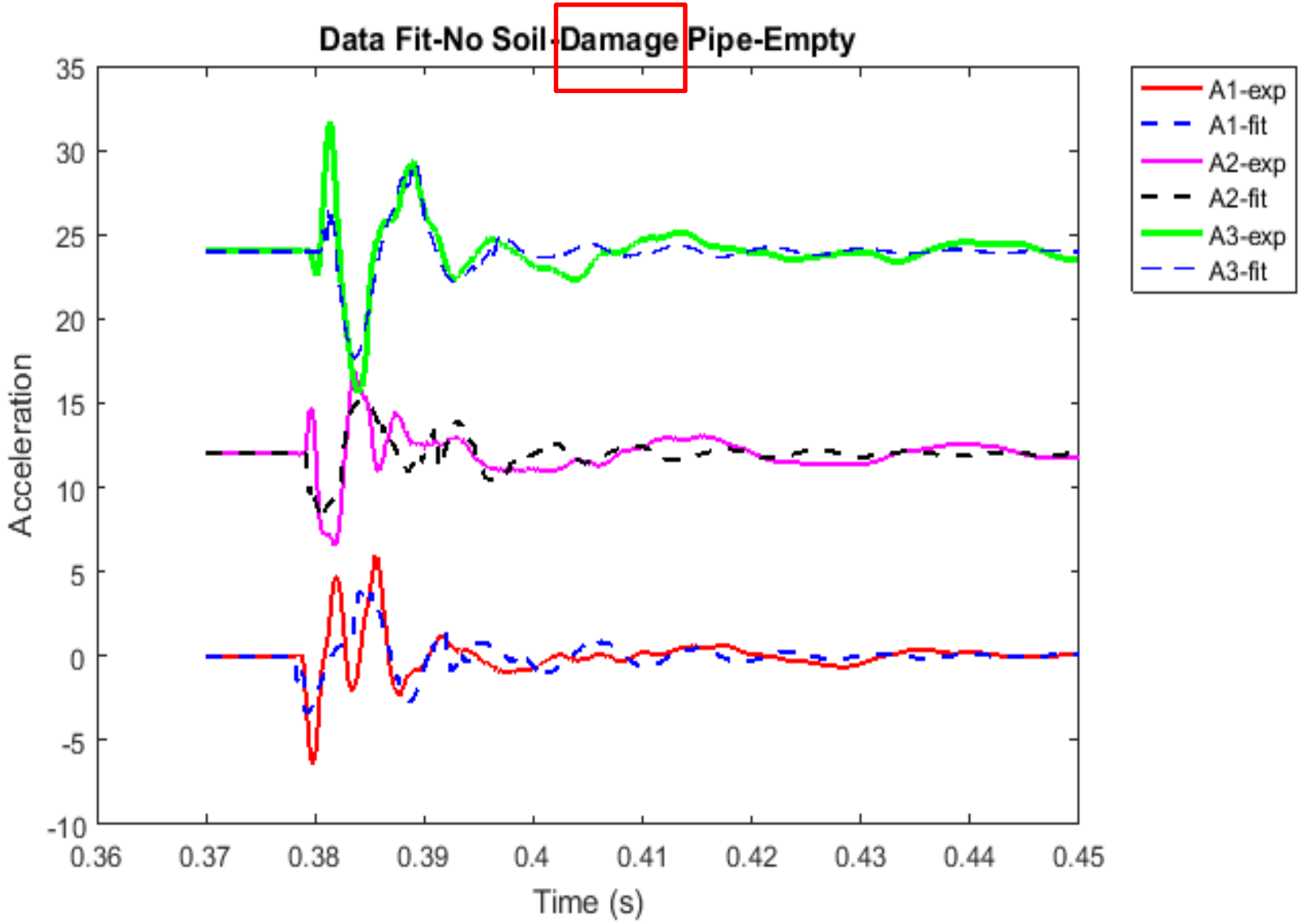
Data fit



Results

Damage pipe

Data fit



Results

Comparison

Resolved values

Model parameter	Healthy pipe	Corroded pipe
ω_1 (Hz)	248.75	214.12
ω_2 (Hz)	154.25	129.51
ω_3 (Hz)	53.50	59.21
v_1 (m/s)	276.25	237.00
v_2 (m/s)	151.50	150.17
k (s^{-1})	34.47	12.39
k_2 (s^{-1})	199.75	191.03
k_3 (s^{-1})	25.68	26.24

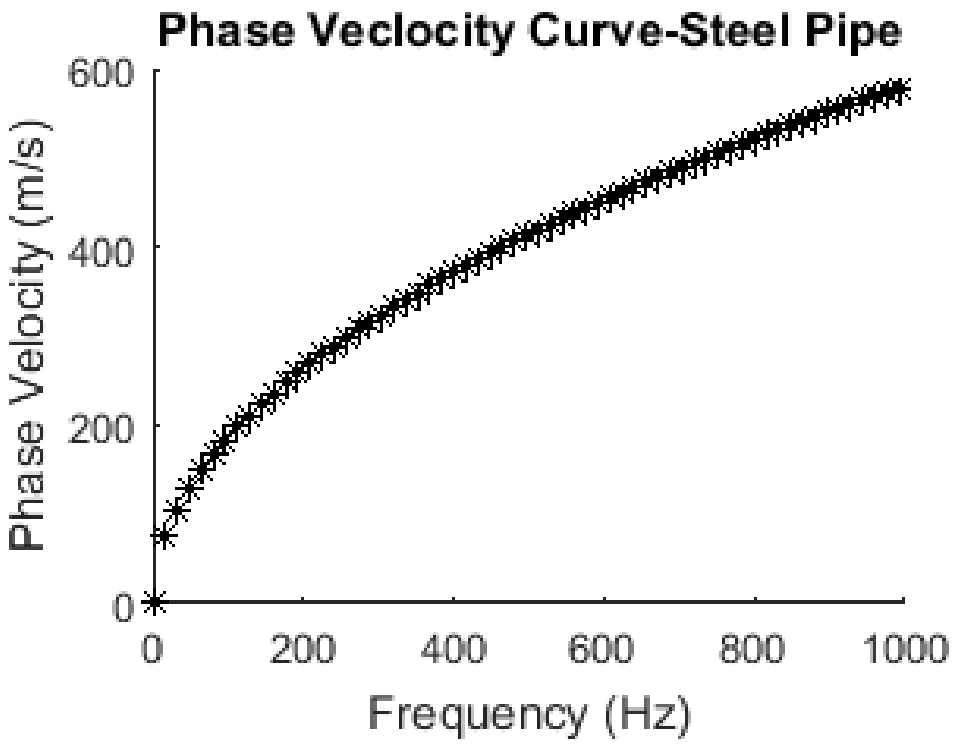
Results

Velocity comparison

Phase velocity of propagating wave

- Propagative wave's frequency ω_1 = 248.75 Hz
- Error: 6 %

Parameter	Velocity from model	Theoretical velocity
v_1 (m/s)	276.25	293



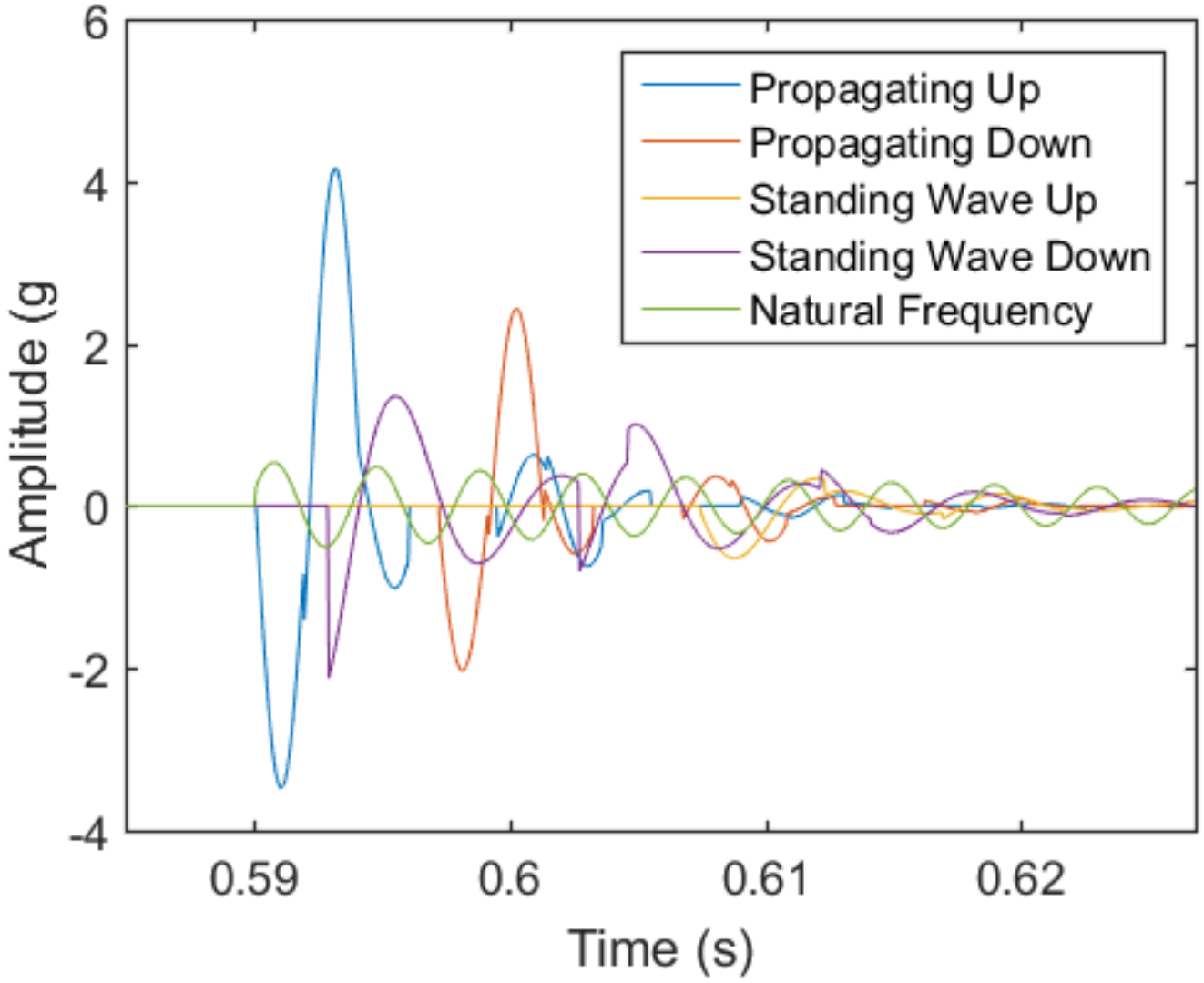
F(1,1) branch:

E=200 GPa
rho=7800 kg/m^3
nu=0.29
ri=14 mm
t=3mm

Results

Separation of wave

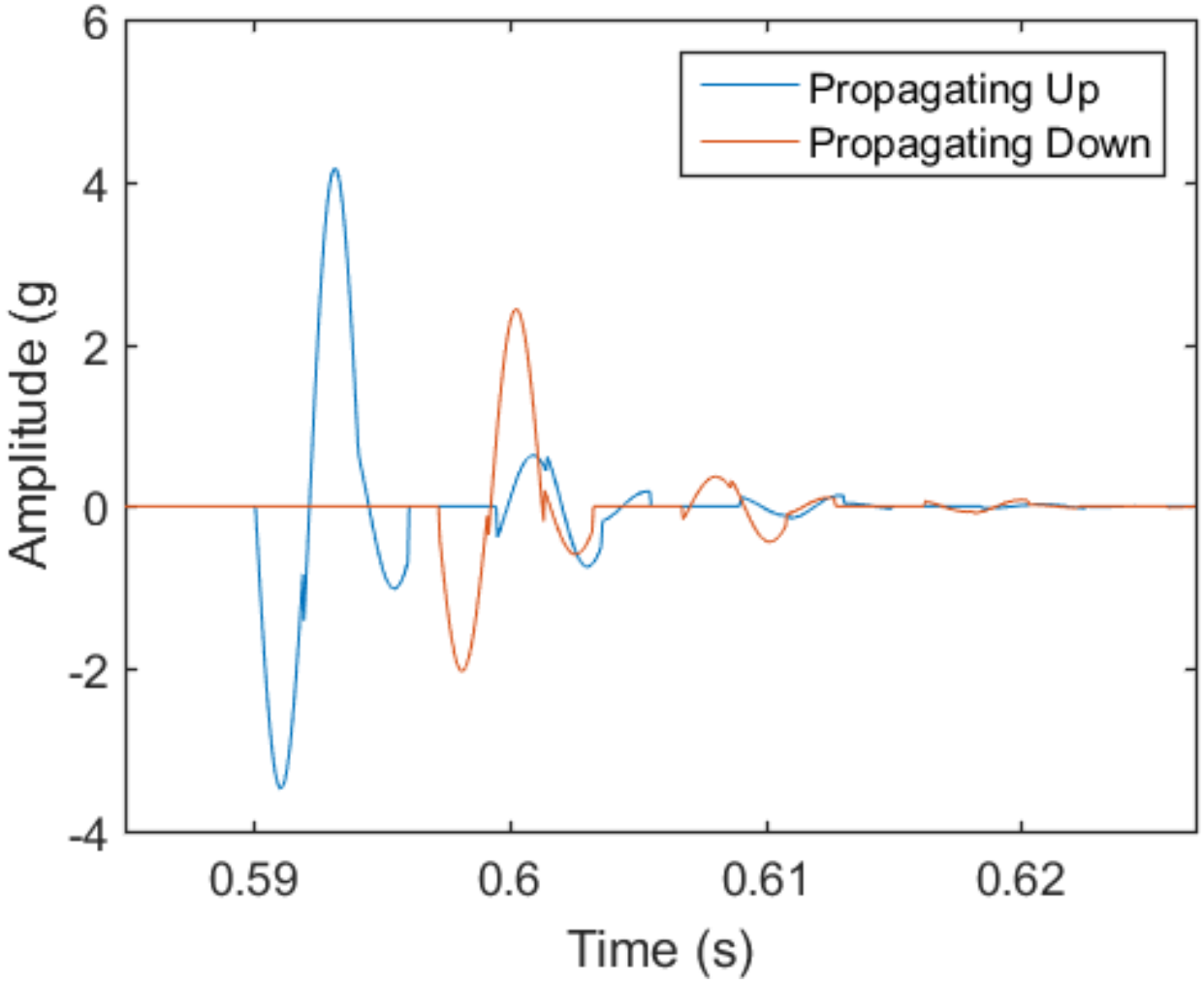
Reflection peak from boundary



Reflection peak from boundary: Propagating wave

Results

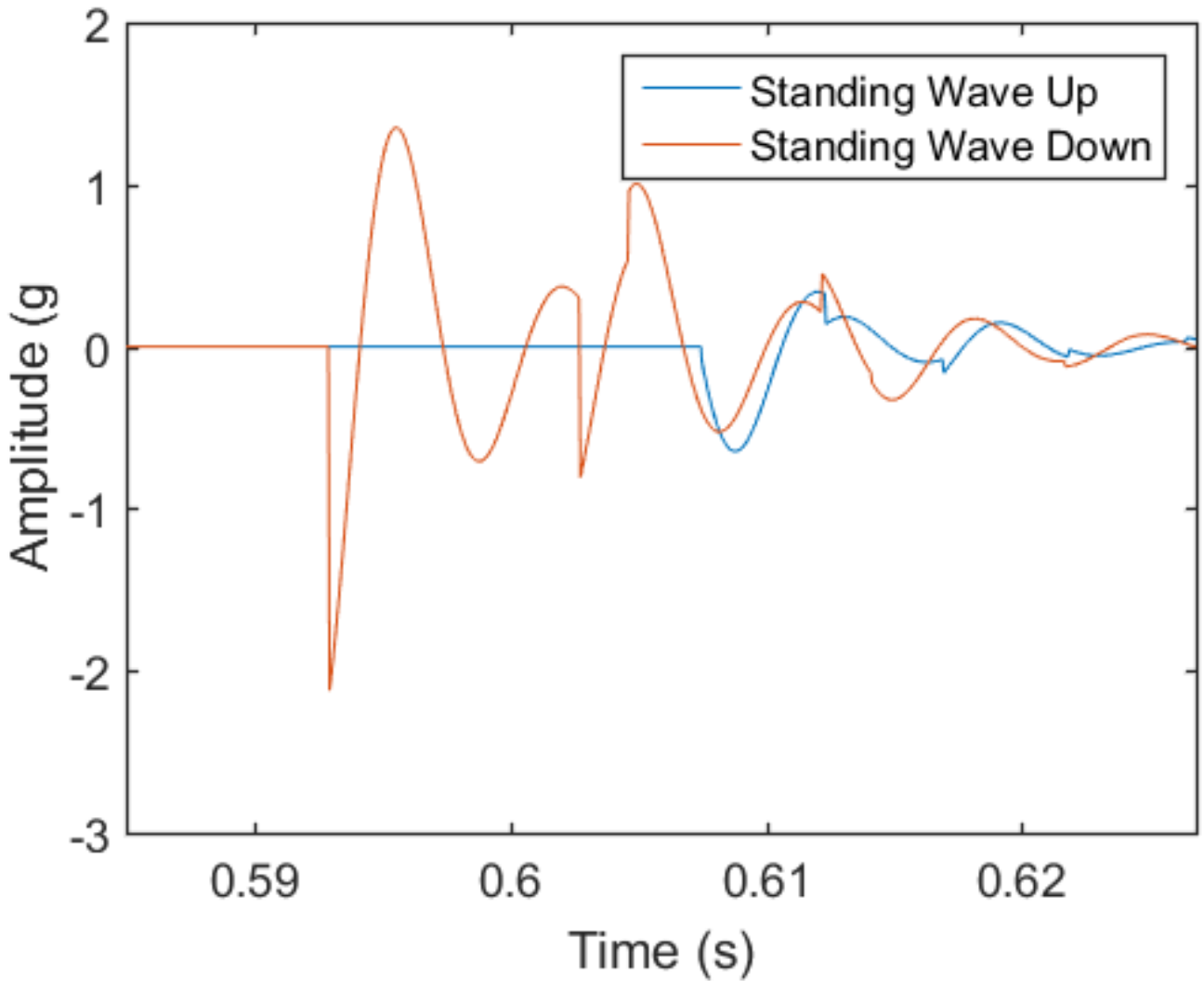
Separation of wave



Reflection peak from boundary: Standing wave

Results

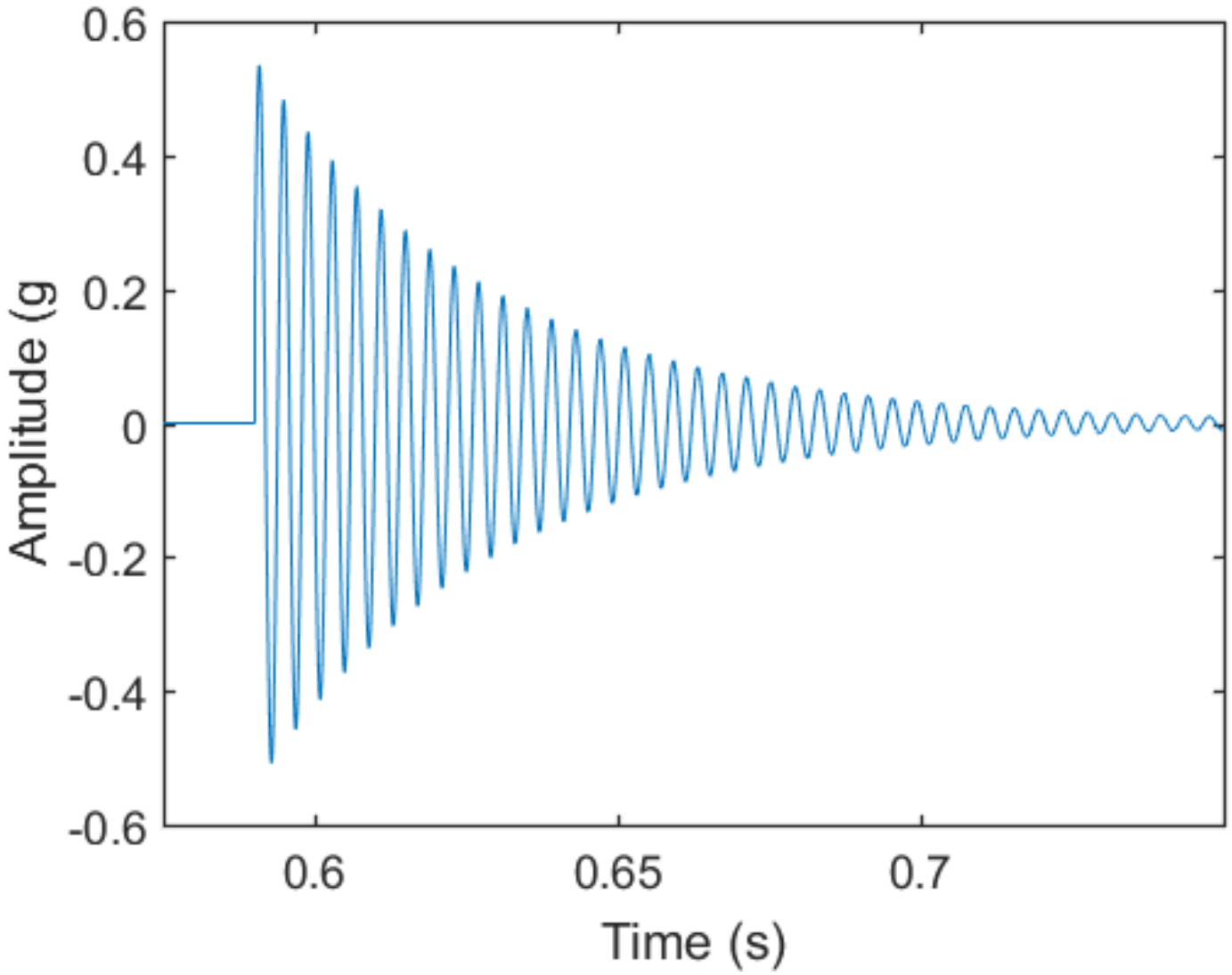
Separation of wave



Results

Separation of wave

Natural frequency



Drawbacks of this approach

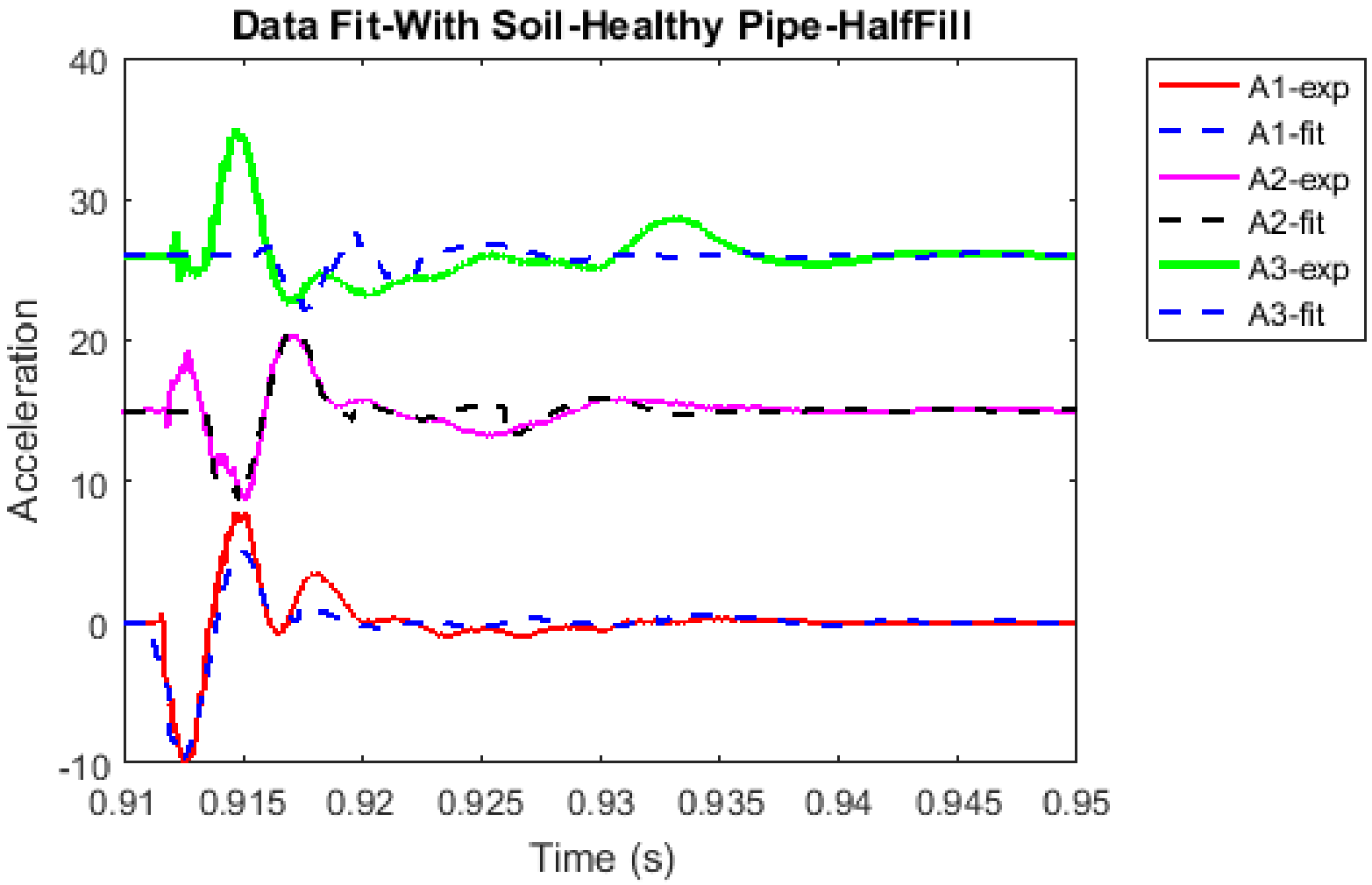
Nothings perfect

- Long computational time
  - Potential to have a converged result in as little as 50,000 generations (2-3 hours), but typically to ensure convergence it is recommended to run for 400,000-500,000 generations (30+ hours).
- Dependence on having the right input wavelet
  - Here we have assumed a basic sinusoidal wave for simplicity, but this is a major flaw in the current approach
    - This is proof-of-concept work
- Thorough understanding of the physics to accurately build the right model
  - Still some work here required to refine the current model as shown, there are still inconsistencies between the signal and the model.
  - It is possible this could be due solely to the input wavelet, but there may also be other critical things missed in the model development

Real steel pipe - Healthy

Drawbacks

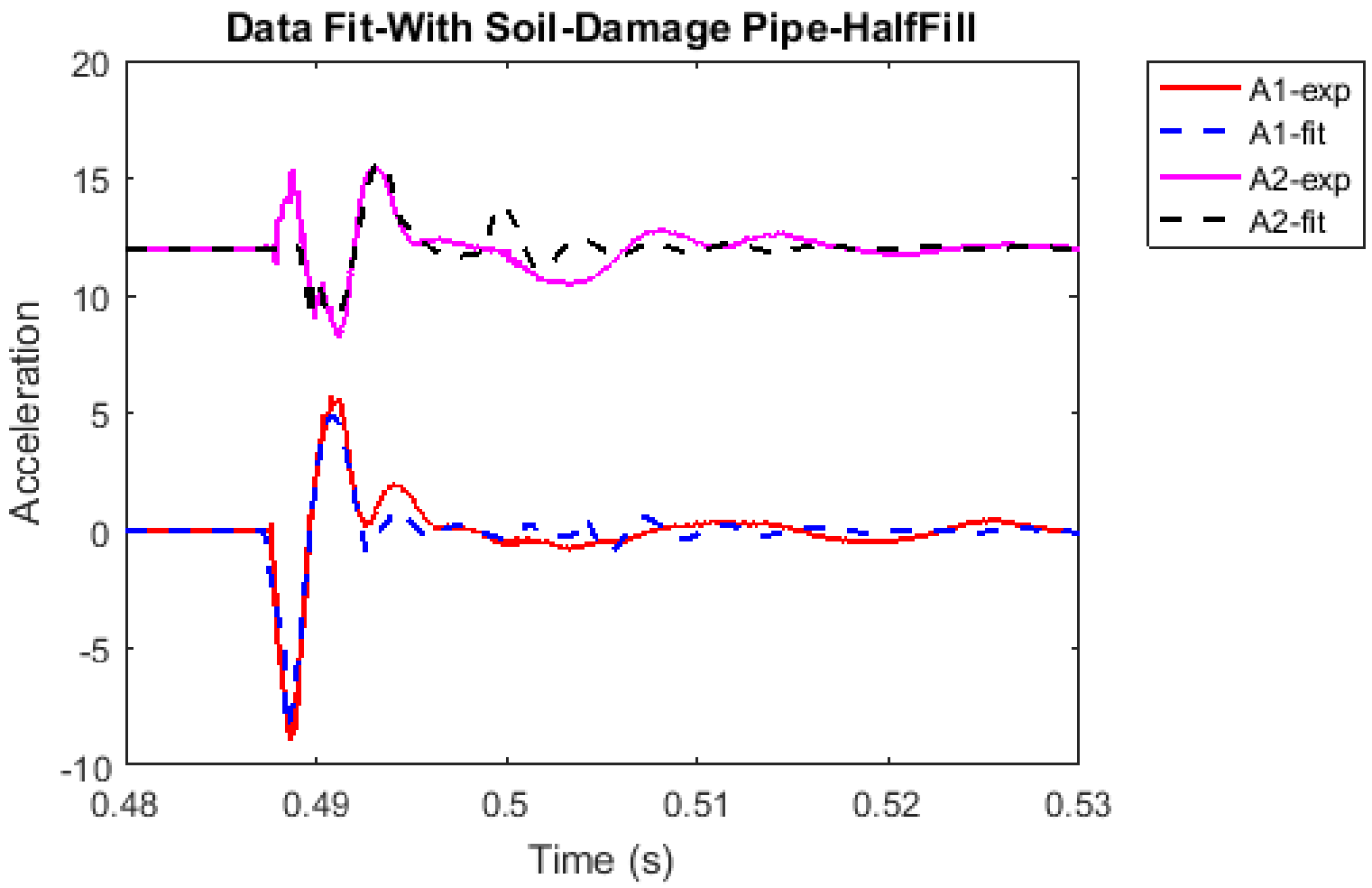
Real boundary condition



Real steel pipe - Corroded

Drawbacks

Nothings perfect



Summary

- Simple physical models are capable of fitting to impact signals from hollow pipes
- It is possible to accurately determine the propagating wave frequency and velocity in addition to the natural frequency from the time-series
  - Therefore this technique has strong applications in NDE
- Owing to the summative nature of the model, it is possible to split the signal into fundamental components
  - Natural frequency
  - Both propagating waves
  - Standing waves