# Sensitivity-Based Model Updating of Cable-Stayed Bridges Considering Monitoring Data Variability

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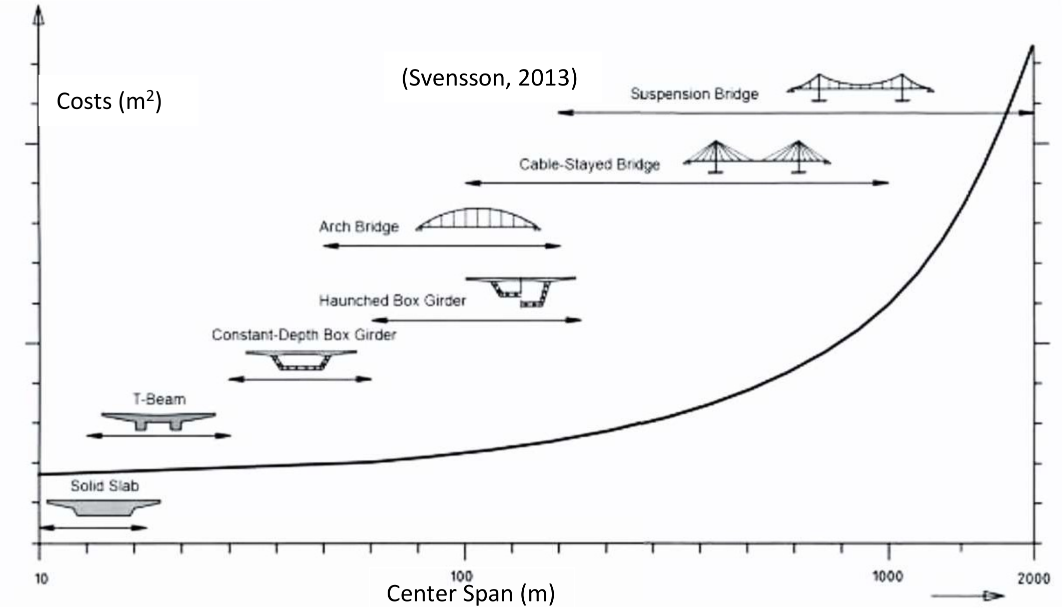
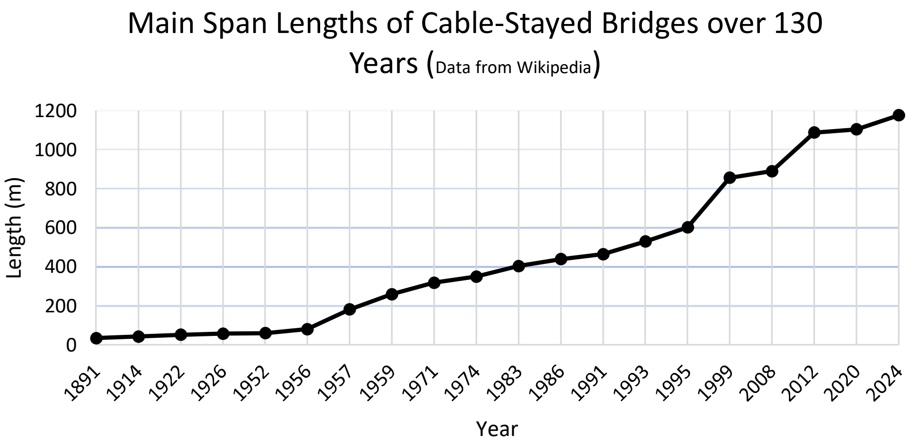
# Outline

1. Background
2. Case Study Bridge and SHM Data
3. Initial Finite Element Model
4. Finite Element Model Updating

# 1. Background – Bridge Failures

# 1. Background - Cable-Stayed Bridges




- Globally, of the total bridge construction in 2019, investment in cable-stayed bridges ≈ truss + arch + suspension bridges.
- Evolving design and construction technologies mean spans now reach over 1000m.
- Ease and speed of construction – the same flow of forces is present during free-cantilever construction as after completion.
- Much greater stiffness than suspension bridges – deformations of the deck are reduced, critical wind speed for the onset of flutter is higher.

# 1. Background – SHM of Cable-Stayed Bridges

- Hong Kong
- Kap Shui Mun Bridge
- Ting Kau Bridge
- Stonecutters Bridge
- Shenzhen Bay Bridge

- China
- Over 80 cable supported bridges with SHM systems
- Hutong Yangtze River Bridge (2nd longest cable-stayed bridge)
- Sutong Yangtze River Bridge (3rd longest cable-stayed bridge)
- Hong Kong-Zhuhai-Macao Bridge (55km bridge/tunnel system)

- South Korea
- Seohae Bridge
- Jindo Bridge
- Hwamyeong Bridge
- Incheon Bridge

- Vietnam
- Phu My Bridge
- Can Tho Bridge
- Rach Mieu Bridge
- Bay Chay Bridge
- Kien Bridge

- Japan
- Tatara Bridge (longest cable-stayed bridge in Japan)

- France
- Millau Viaduct (tallest cable-stayed bridge in the world)

- Sweden/Denmark
- Oresund Bridge (longest road/rail bridge in Europe)

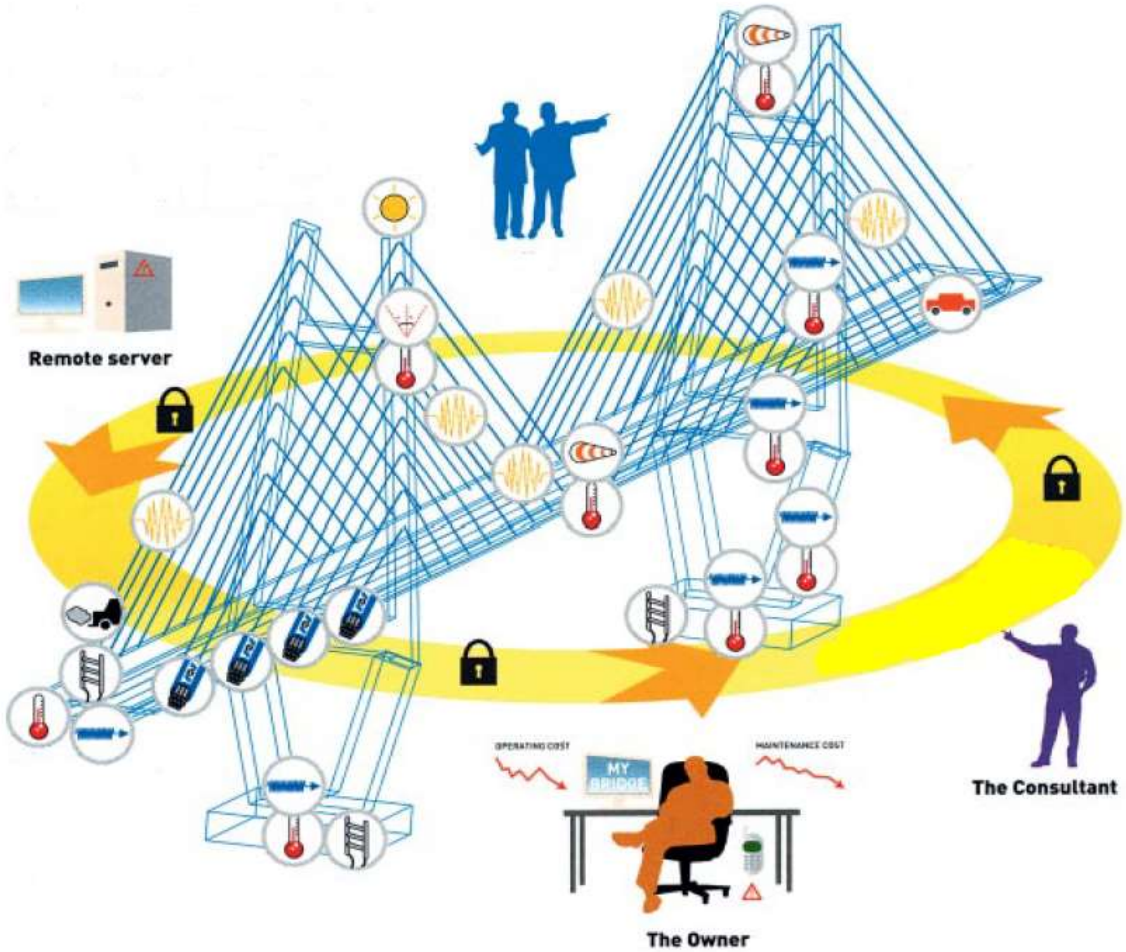
- Scotland
- Queensferry Crossing (world's longest three tower cable-stayed bridge)

- USA
- Arthur Ravenel Jr Bridge (3rd longest in Western hemisphere)

- Panama
- Atlantic Bridge

- Portugal/Spain
- Guadiana International Bridge

# 1. Background ➡ SHM Monitoring Data

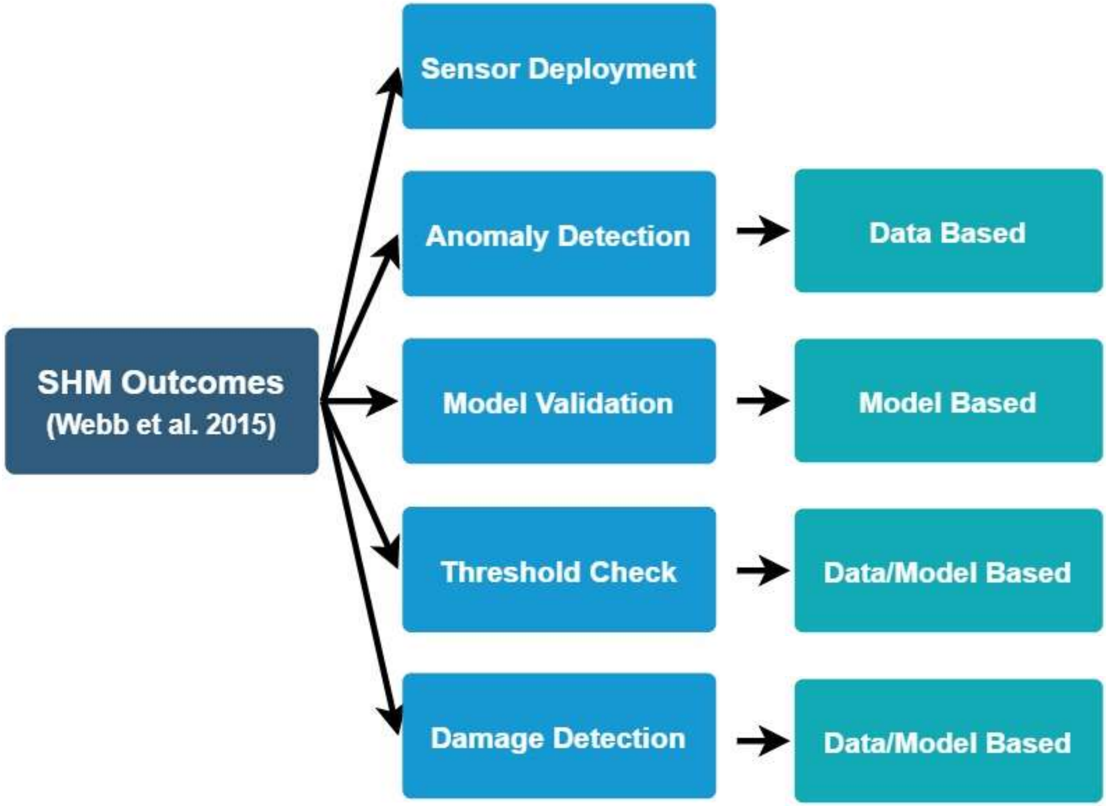



(Svensson, 2013)

<u>Data Measurements</u>

- Strain
- Acceleration
- Displacement
- Inclination
- Temperature
- Wind speed
- Humidity
- Weigh-in-motion
- Corrosion

# 1. Background - SHM Outcomes




Data Based

- Examine changes in data features
- Use of data-driven algorithms
- Statistical pattern recognition/machine learning
- High confidence in the data
- Sufficient numbers and locations of sensors to offer level of completeness

Model Based

- Data used with numerical model of structure
- Insight into physical processes
- Can simulate behaviour under various conditions
- Useful when sensors are limited, model have offer complete picture
- Uncertainties and assumptions introduced when modelling

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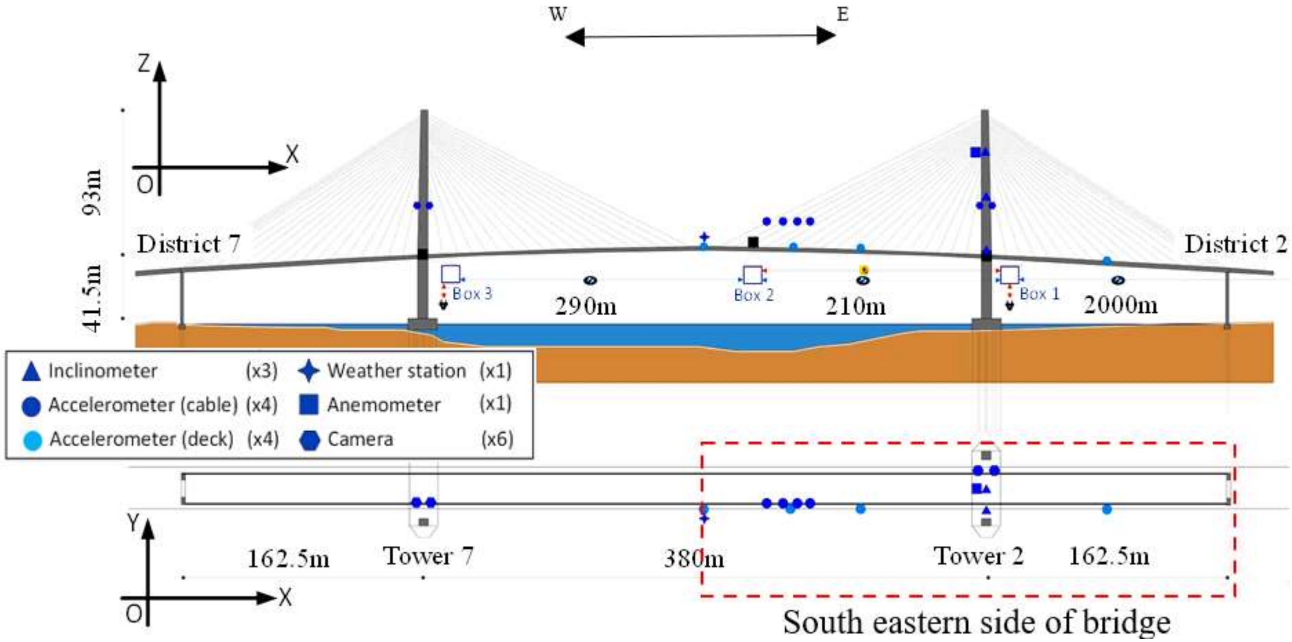
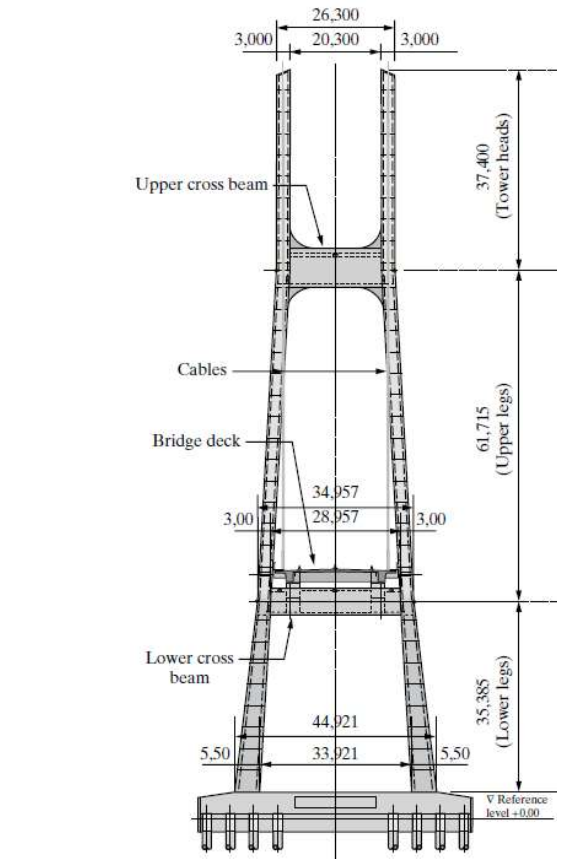
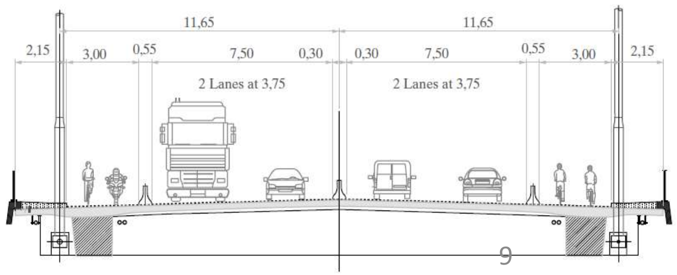
# 2. Case Study - Phu My Bridge




<u>Features</u>

- Concrete Bridge
- 380m main span
- 162.5m side spans
- 134.5m high towers
- 27.5m wide deck
- 144 stay cables

<u>SHM System</u>

- 3 Inclinometers
- 8 Accelerometers
- 6 Cameras
- 1 Weather station
- 1 Anemometer

<u>Dates</u>

- Opened Sept. 2009
- SHM installed 2019

# 2. Phu My Bridge Sensors and Data Sample

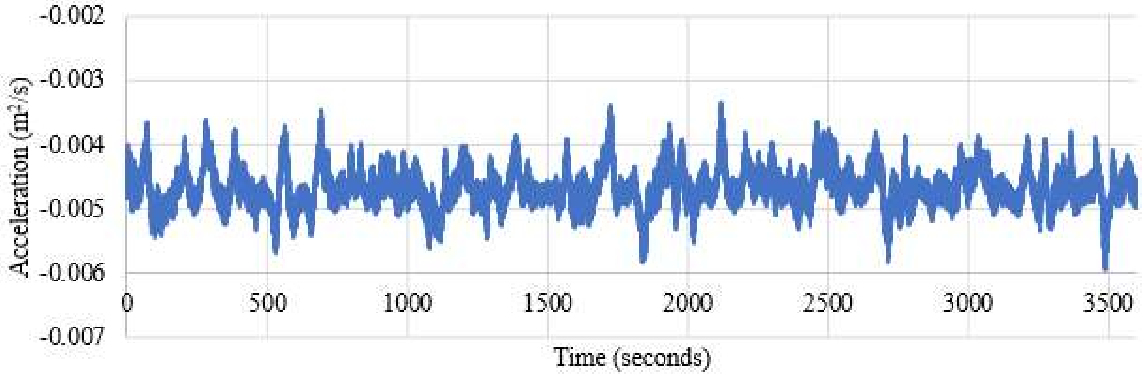
Phu My Bridge SHM Sensors

| Type | Quantity | Sampling Rate | Parameters Monitored | Data Storage Format |
|---|---|---|---|---|
| Inclinometer | 3 | 50 Hz | Tilt of tower | CSV |
| Accelerometer | 8 | 50 Hz | Accelerations of lateral and vertical motions | CSV |
| Weather station | 1 | 1 Hz | Air temperature, humidity, pressure, wind speed and direction | CSV |
| Anemometer | 1 | 1 Hz | Wind speed and direction | CSV |

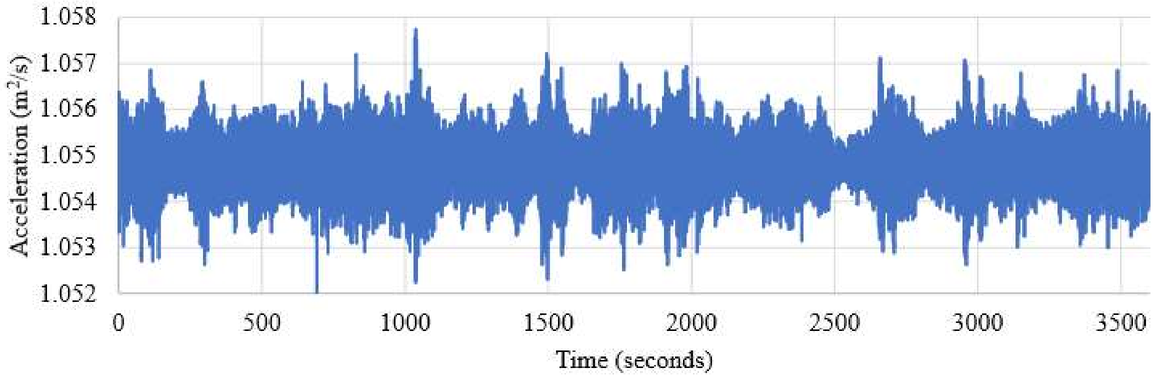
Data Sample from SHM System

| Dates (inclusive) | No. of days | No. of one-hour data sets available |
|---|---|---|
| 23-30 April 2020 | 8 | 191 |
| 29 July – 04 August 2020 | 7 | 168 |
| 25-31 August 2020 | 7 | 168 |
| 24-30 September 2020 | 7 | 168 |
| 25-31 October 2020 | 7 | 168 |
| 24-30 November 2020 | 7 | 168 |
| Total | 43 | 1031 |

# 2. Phu My Bridge – Raw Data

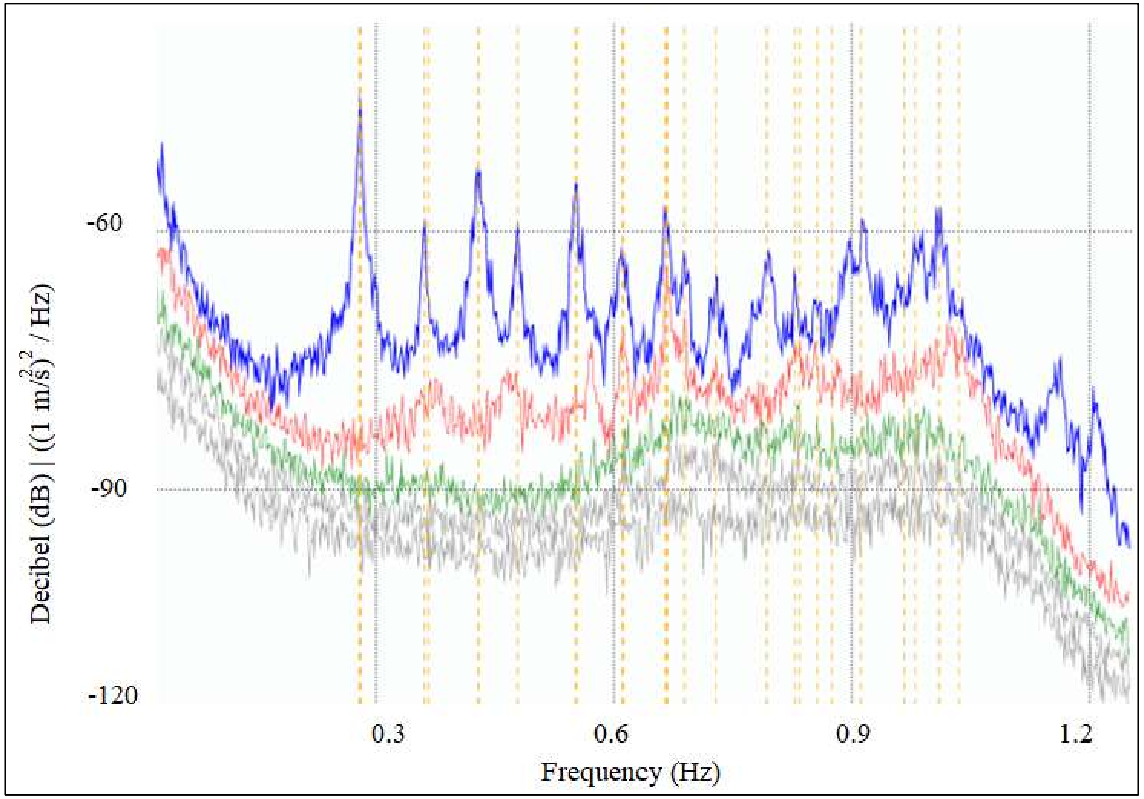



Lateral direction time series acceleration data – main span (3-4am 28 April 2020)



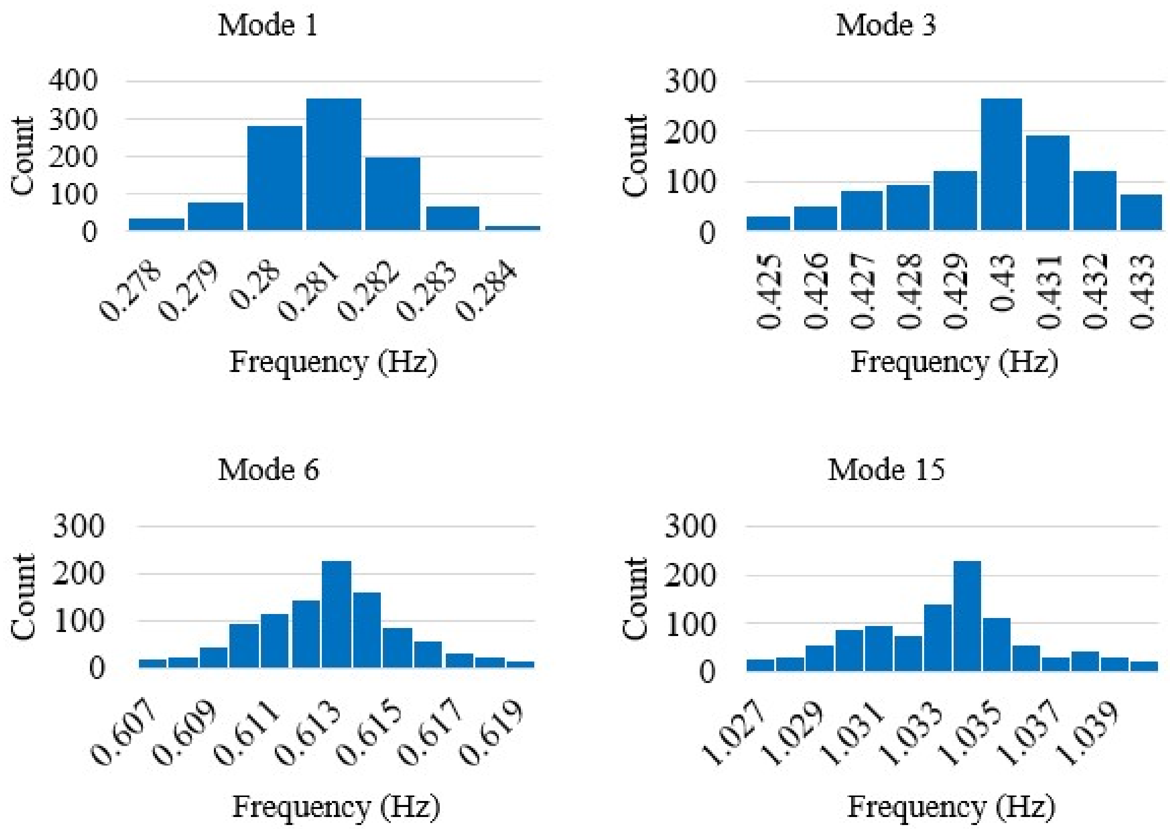

Vertical direction time series acceleration data – main span (2-3am 04 August 2020)

# 2. Operational Modal Analysis



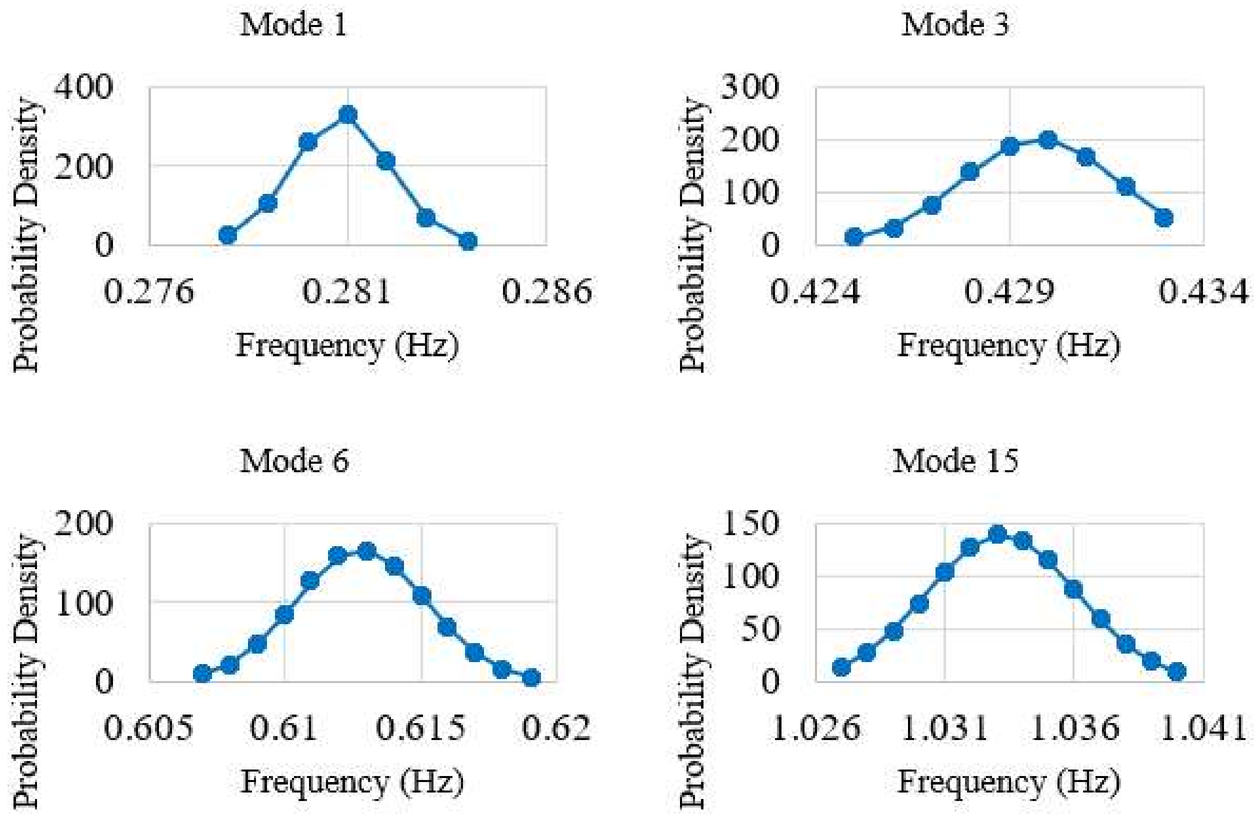

Frequency peaks of singular value decomposition (SVD) identified using Enhanced Frequency Domain Decomposition method.

# 2. Operational Modal Analysis - Results



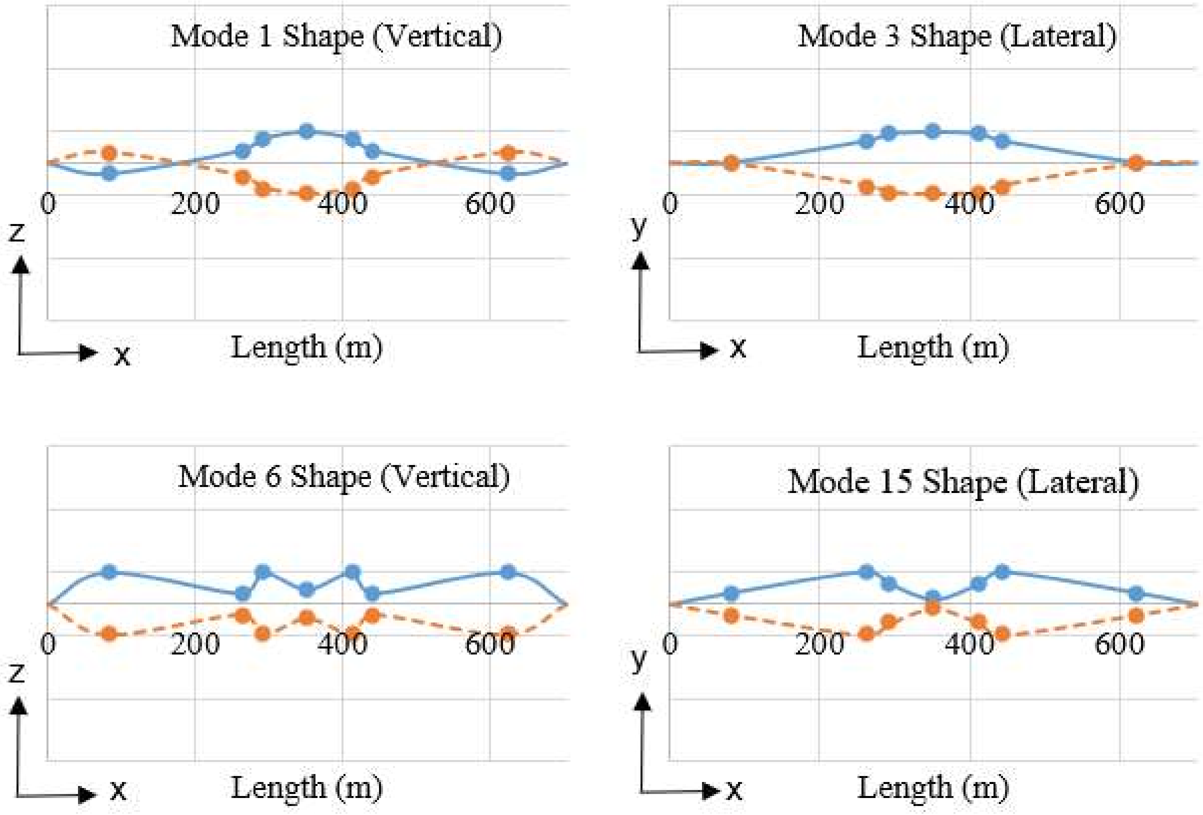

Histograms of selected natural frequency ranges identified from accelerometer data.

# 2. Operational Modal Analysis - Results




Probability density distributions identified from accelerometer data.

# 2. Operational Modal Analysis - Results




Selected mode shapes identified from accelerometer data.

# 2. Operational Modal Analysis - Results

Frequency and mode shapes identified from OMA of deck accelerometers.

| **Identified Mode No.** | **Frequencies (Hz)** | | | | **Deck Mode Shape** |
|---|---|---|---|---|---|
| | Mean | Min / Max | Range | Std. Deviation | |
| 1 | 0.281 | 0.278/ 0.284 | 0.006 | 0.0012 | Vertical |
| 2 | 0.362 | 0.358 / 0.366 | 0.008 | 0.0018 | Vertical |
| 3 | 0.430 | 0.425 / 0.433 | 0.008 | 0.0020 | Lateral |
| 4 | 0.479 | 0.475 / 0.483 | 0.008 | 0.0018 | Vertical |
| 5 | 0.555 | 0.551 / 0.559 | 0.008 | 0.0022 | Vertical |
| 6 | 0.613 | 0.607 / 0.619 | 0.012 | 0.0024 | Vertical |
| 7 | 0.668 | 0.663 / 0.672 | 0.009 | 0.0021 | Vertical |
| 8 | 0.731 | 0.724 / 0.737 | 0.012 | 0.0030 | Vertical |
| 9 | 0.796 | 0.789 / 0.802 | 0.013 | 0.0030 | Vertical |
| 10 | 0.854 | 0.850 / 0.859 | 0.009 | 0.0023 | Lateral |
| 11 | 0.895 | 0.890 / 0.900 | 0.010 | 0.0025 | Vertical |
| 12 | 0.941 | 0.935 / 0.947 | 0.012 | 0.0030 | Vertical |
| 13 | 0.984 | 0.979 / 0.989 | 0.010 | 0.0023 | Vertical |
| 14 | 1.013 | 1.010 / 1.018 | 0.008 | 0.0021 | Vertical |
| 15 | 1.033 | 1.027 / 1.040 | 0.013 | 0.0030 | Lateral |
| 16 | 1.161 | 1.150 / 1.169 | 0.019 | 0.0049 | Vertical |
| 17 | 1.211 | 1.202 / 1.217 | 0.015 | 0.0035 | Vertical |

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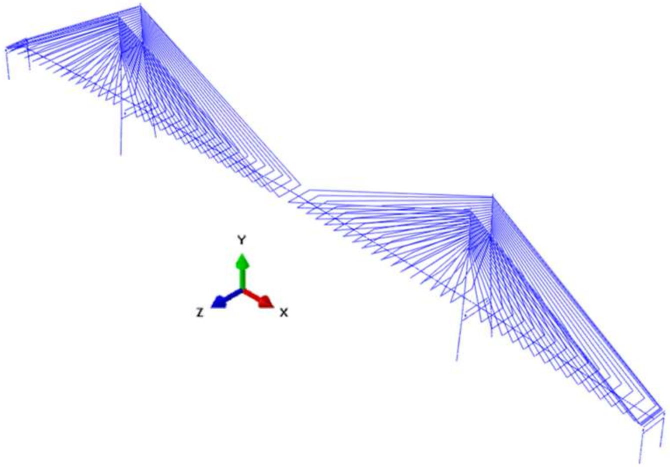
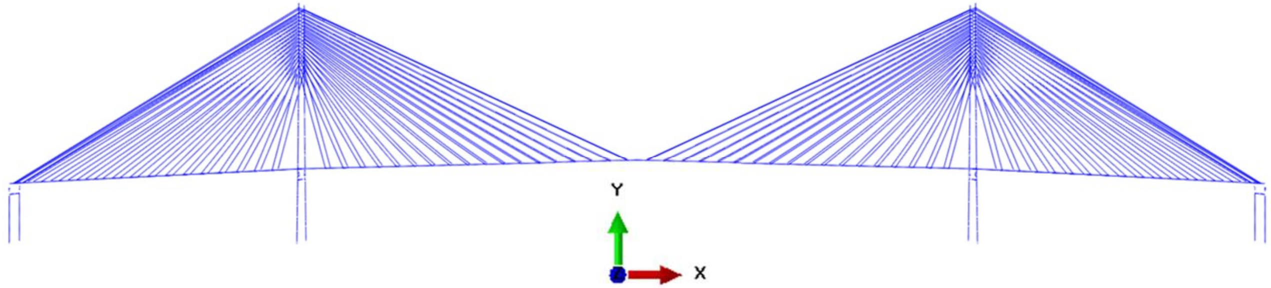
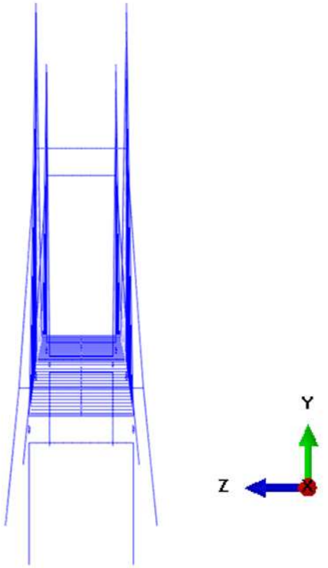
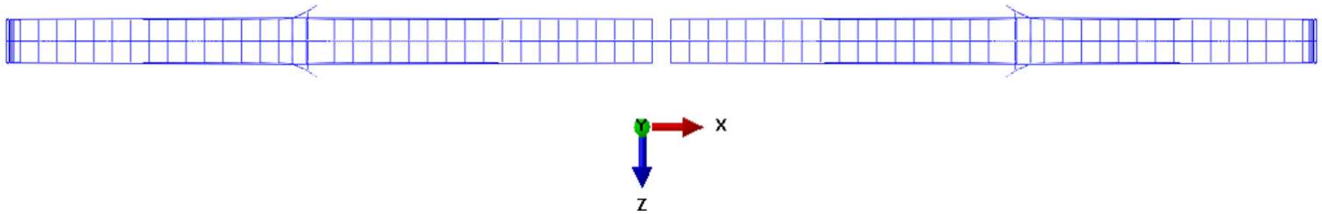
# 3. Phu My Bridge Initial FE Model



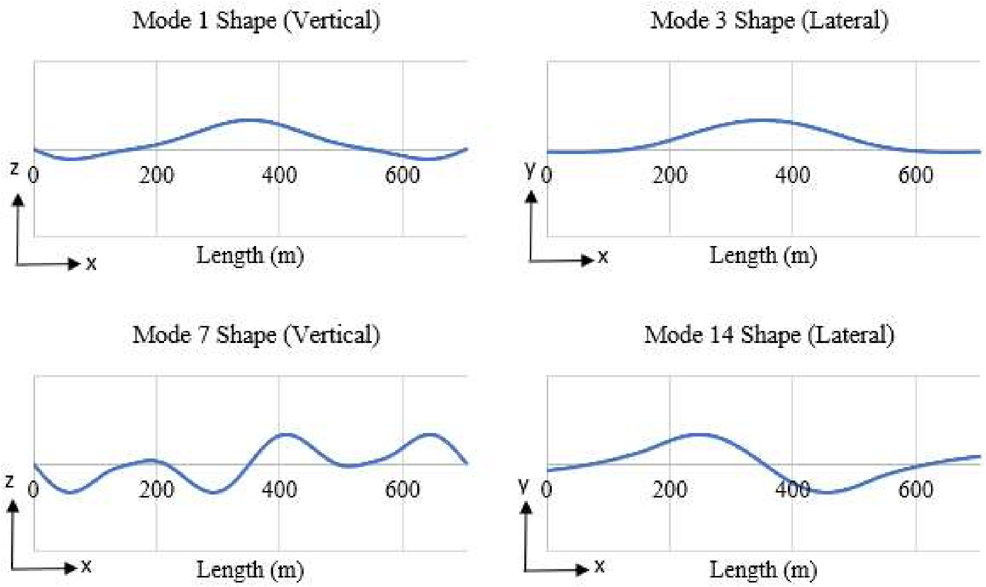



# 3. Phu My Bridge Initial FE Model Inputs

| Index | Parameters | Initial Estimation |
|---|---|---|
| Geometric Parameters | | |
| G1 | Cross sectional area of tower lower leg | 35 $m^2$ |
| G2 | Cross sectional area of tower side leg | 18 $m^2$ |
| G3 | Cross sectional area of tower upper leg | 15 $m^2$ |
| G4 | Cross sectional area of tower lower cross beam | 30 $m^2$ |
| G5 | Cross sectional area of tower upper cross beam | 25 $m^2$ |
| G6 | Cross sectional area of end pier leg | 15 $m^2$ |
| G7 | Cross sectional area of end pier cross beam | 6 $m^2$ |
| G8 | Cross sectional area of girder | 23 $m^2$ |
| G9 | Moment of inertia of girder | 21 $m^4$ |
| G10 | Torsional inertia of girder | 58 $m^4$ |
| G11 | Cross sectional area of stay cables | Varied |
| Non-structural Parameters | | |
| NS12 | Non-structural girder mass per unit length | 15000 kg/m |
| Material Parameters | | |
| M13 | Concrete strength of towers and piers | 40 MPa |
| M14 | Concrete strength of girder | 50 MPa |
| M15 | Elastic modulus of stay cables | 195 GPa |
| M16 | Density of concrete | 2500 kg/$m^3$ |
| M17 | Density of stay cables | 7850 kg/$m^3$ |
| M18 | Poisson's ratio of concrete | 0.2 |
| M19 | Poisson's ratio of stay cables | 0.3 |
| M20 | Spring stiffness | 3 x $10^7$ N/m |

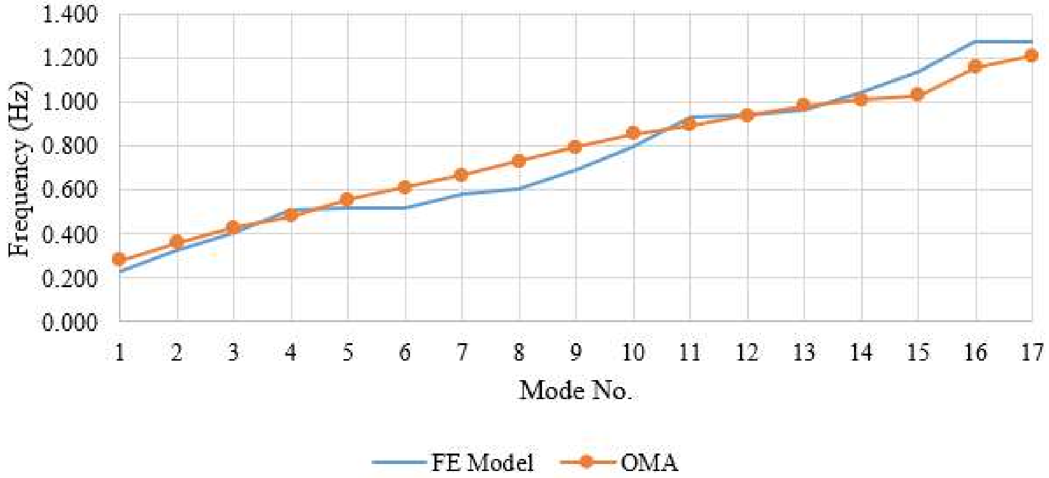
# 3. Phu My Bridge Initial FE Model Results

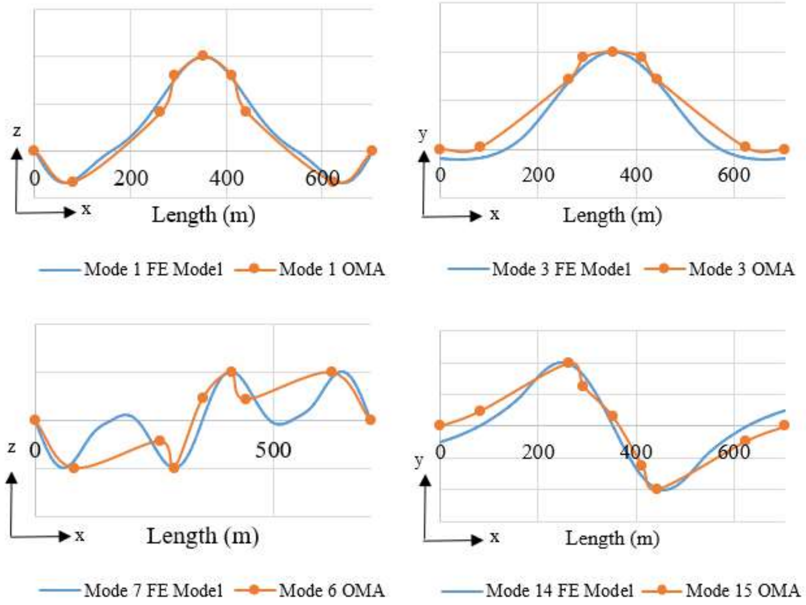
| Mode No. | Frequencies (Hz) | Mode Shape Description |
|---|---|---|
| 1 | 0.233 | Deck - Vertical – Symmetrical |
| 2 | 0.324 | Deck - Vertical – Antisymmetrical |
| 3 | 0.405 | Deck - Lateral – Symmetrical |
| 4 | 0.509 | Deck - Vertical – Symmetrical |
| 5 | 0.516 | Tower – Lateral – Antisymmetrical |
| 6 | 0.519 | Tower – Lateral – Symmetrical |
| 7 | 0.581 | Deck - Vertical – Antisymmetrical |
| 8 | 0.605 | Deck – Vertical - Antisymmetrical |
| 9 | 0.693 | Deck - Vertical – Symmetrical |
| 10 | 0.799 | Deck – Torsion |
| 11 | 0.936 | Pier – Lateral – Antisymmetrical |
| 12 | 0.941 | Pier – Lateral – Symmetrical |
| 13 | 0.968 | Deck - Vertical – Antisymmetrical |
| 14 | 1.047 | Deck - Lateral – Antisymmetrical |
| 15 | 1.138 | Deck - Vertical – Symmetrical |
| 16 | 1.277 | Deck - Vertical – Antisymmetrical |
| 17 | 1.281 | Tower Upper Legs – Lateral - Symmetrical |



# 3. Phu My Bridge Initial FE Model Result Comparison

| Mode No. | Initial FE (IFE) Model (Hz) | OMA (Hz) | Percentage Difference (IFE-OMA)/OMA | IFE Mode Shape Description | OMA Deck Mode Shape Description |
|---|---|---|---|---|---|
| 1 | 0.233 | 0.281 | -17.08% | Deck - Vertical - Symmetrical | Vertical |
| 2 | 0.324 | 0.362 | -10.50% | Deck - Vertical - Antisymmetrical | Vertical |
| 3 | 0.405 | 0.430 | -5.81% | Deck - Lateral - Symmetrical | Lateral |
| 4 | 0.509 | 0.479 | +6.26% | Deck - Vertical - Symmetrical | Vertical |
| 5 | 0.516 | 0.555 | -7.03% | Tower - Lateral - Antisymmetrical | Vertical |
| 6 | 0.519 | 0.613 | -15.33% | Tower - Lateral - Symmetrical | Vertical |
| 7 | 0.581 | 0.668 | -13.02% | Deck - Vertical - Antisymmetrical | Vertical |
| 8 | 0.605 | 0.731 | -17.24% | Deck - Vertical - Antisymmetrical | Vertical |
| 9 | 0.693 | 0.796 | -12.94% | Deck - Vertical - Symmetrical | Vertical |
| 10 | 0.799 | 0.854 | -6.44% | Deck – Torsion | Lateral |
| 11 | 0.936 | 0.895 | +4.58% | Pier - Lateral - Antisymmetrical | Vertical |
| 12 | 0.941 | 0.941 | 0.00% | Pier - Lateral - Symmetrical | Vertical |
| 13 | 0.968 | 0.984 | -1.63% | Deck - Vertical - Antisymmetrical | Vertical |
| 14 | 1.047 | 1.013 | +3.36% | Deck - Lateral - Antisymmetrical | Vertical |
| 15 | 1.138 | 1.033 | +10.16% | Deck - Vertical - Symmetrical | Lateral |
| 16 | 1.277 | 1.161 | +9.99% | Deck - Vertical - Antisymmetrical | Vertical |
| 17 | 1.281 | 1.211 | +5.78% | Tower Upper Legs - Lateral - Symmetrical | Vertical |



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# 4. Model Updating – Objective Function

Original FE model input parameters

$$\boldsymbol{P}_n = [p_{i,n} \,|i = 1, 2, 3, \dots \dots, n_{pn}]^{\mathrm{T}}$$

Original FE model results

$$\boldsymbol{\Lambda}_n = [\lambda_{i,n} \,|i = 1, 2, 3, \dots \dots, n_{\lambda n}]^{\mathrm{T}}$$

Unknown 'objective' parameters

$$\boldsymbol{P}_u = [p_{i,u} \,|i = 1, 2, 3, \dots \dots, n_{pu}]^{\mathrm{T}}$$

Measured structure results

$$\boldsymbol{\Lambda}_m = [\lambda_{i,m} \,|i = 1, 2, 3, \dots \dots, n_{\lambda m}]^{\mathrm{T}}$$

$$\boldsymbol{\Lambda}_m - \boldsymbol{\Lambda}_n = \boldsymbol{S}(\boldsymbol{P}_u - \boldsymbol{P}_n)$$

$$\boldsymbol{\delta\Lambda} = \boldsymbol{S\delta P}$$

$$J = (\boldsymbol{S\delta P} - \boldsymbol{\delta\Lambda})^{\mathrm{T}} \boldsymbol{W}_E (\boldsymbol{S\delta P} - \boldsymbol{\delta\Lambda}) + \boldsymbol{\delta P}^{\mathrm{T}} \boldsymbol{W}_P \boldsymbol{\delta P}$$

$W_E$ = weighting matrix reflecting measurement uncertainty / variability (inverse of the data variance)

$W_P$ = weighting matrix reflecting parameter uncertainty / variability

# 4. Model Updating – Selection of Updating Parameters

$$S_{ij} = \left(\frac{\delta\lambda_{i,n}}{\delta P_{j,n}}\right)\cdot\left(\frac{P_{j,n}}{\lambda_{i,n}}\right)$$

| **Sensitivity rank** | **Index** | **Parameters** | **Sensitivity effect** |
|---|---|---|---|
| 1 | M16 | Density of concrete | - |
| 2 | M15 | Elastic modulus of stay cables ($E_o$ in Eq. (12)) | + |
| 3 | G8 | Cross sectional area of girder | - |
| 4 | G11 | Cross sectional area of stay cables | + |
| 5 | G2 | Cross sectional area of tower side leg | + |
| 6 | G1 | Cross sectional area of tower lower leg | +/- |
| 7 | M13 | Concrete strength of towers and piers | + |
| 8 | G9 | Moment of inertia of girder | + |
| 9 | M14 | Concrete strength of girder | + |
| 10 | NS12 | Non-structural girder mass per unit length | - |
| 11 | G3 | Cross sectional area of tower upper leg | +/- |
| 12 | M17 | Density of stay cables | - |
| 13 | G4 | Cross sectional area of tower lower cross beam | + |
| 14 | G5 | Cross sectional area of tower upper cross beam | +/- |
| 15 | G10 | Torsional inertia of girder | + |
| 16 | G6 | Cross sectional area of end pier leg | +/- |
| 17 | M20 | Spring stiffness | - |
| 18 | G7 | Cross sectional area of end pier cross beam | No effect |
| 19 | M18 | Poisson's ratio of concrete | No effect |
| 20 | M19 | Poisson's ratio of stay cables | No effect |

# 4. Model Updating – Parameters for Updating

| Index | Parameter |
|---|---|
| M16 | Density of concrete |
| M15 | Elastic modulus of stay cables |
| G8 | Cross sectional area of girder |
| M13 | Concrete strength of towers and piers |
| G9 | Moment of inertia of girder |
| M14 | Concrete strength of girder |
| NS12 | Non-structural girder mass per unit length |

| Index | Parameter | Initial Value | Range of Values |
|---|---|---|---|
| M16 | Density of concrete | 2500 $kg/m^3$ | 2400-2600 $kg/m^3$ |
| M15 | Elastic modulus of stay cables ($E_o$) ($E_{eq}$ was calculated for each cable based on $E_o$) | 195 GPa | 190-199 GPa |
| G8 | Cross sectional area of girder | 23 $m^2$ | 15-30 $m^2$ |
| M13 | Concrete strength of towers and piers | 40 MPa | 30-60 MPa |
| G9 | Moment of inertia of girder | 21 $m^4$ | 15-30 $m^4$ |
| M14 | Concrete strength of girder | 50 MPa | 40-65 MPa |
| NS12 | Non-structural girder mass per unit length | 15000 kg/m | 14000-16000 kg/m |

# 4. Model Updating –Updating Results

| Model Updating Frequency Results (Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Initial FE model | Updated model after each iteration | | | | | | OMA result |
| | 1st | 2nd | 3rd | 4th | 5th | 6th | |
| 0.233 | 0.262 | 0.271 | 0.279 | 0.286 | 0.287 | 0.288 | 0.281 |
| 0.324 | 0.337 | 0.344 | 0.345 | 0.348 | 0.350 | 0.352 | 0.362 |
| 0.405 | 0.418 | 0.429 | 0.435 | 0.437 | 0.439 | 0.441 | 0.430 |
| 0.509 | 0.544 | 0.551 | 0.572 | 0.580 | 0.583 | 0.585 | 0.555 |
| 0.581 | 0.595 | 0.601 | 0.606 | 0.610 | 0.613 | 0.613 | 0.613 |
| 0.605 | 0.619 | 0.634 | 0.649 | 0.651 | 0.659 | 0.660 | 0.668 |
| 0.693 | 0.706 | 0.717 | 0.725 | 0.729 | 0.733 | 0.735 | 0.731 |
| 0.799 | 0.837 | 0.855 | 0.869 | 0.878 | 0.885 | 0.888 | 0.895 |
| 0.968 | 0.957 | 0.956 | 0.953 | 0.950 | 0.949 | 0.947 | 0.984 |
| 1.047 | 1.045 | 1.041 | 1.037 | 1.035 | 1.024 | 1.023 | 1.033 |
| 1.138 | 1.123 | 1.112 | 1.108 | 1.101 | 1.097 | 1.096 | 1.161 |
| 1.277 | 1.254 | 1.237 | 1.220 | 1.215 | 1.212 | 1.211 | 1.211 |

| Mode No. (UFE Model) | Updated FE (UFE) Model (Hz) | Mode No. (OMA) | OMA (Hz) | Percentage Difference (UFE-OMA)/OMA | UFE Model Mode Shape Description | OMA Deck Mode Shape Description |
|---|---|---|---|---|---|---|
| 1 | 0.288 | 1 | 0.281 | +2.49% | Deck - Vertical | Vertical |
| 2 | 0.352 | 2 | 0.362 | -2.76% | Deck - Vertical | Vertical |
| 3 | 0.441 | 3 | 0.430 | +2.56% | Deck - Lateral | Lateral |
| 6 | 0.585 | 5 | 0.555 | +5.41% | Deck - Vertical | Vertical |
| 7 | 0.613 | 6 | 0.613 | 0.00% | Deck - Vertical | Vertical |
| 8 | 0.660 | 7 | 0.668 | -1.20% | Deck – Vertical | Vertical |
| 9 | 0.735 | 8 | 0.731 | +0.55% | Deck - Vertical | Vertical |
| 10 | 0.888 | 11 | 0.895 | -0.78% | Deck – Torsion | Vertical |
| 11 | 0.947 | 13 | 0.984 | -3.76% | Deck - Vertical | Vertical |
| 13 | 1.023 | 15 | 1.033 | -0.97% | End Pier - Lateral | Lateral |
| 14 | 1.096 | 16 | 1.161 | -5.60% | Deck - Vertical | Vertical |
| 18 | 1.211 | 17 | 1.211 | 0.00% | Deck - Vertical | Vertical |

| Index | Parameter | Initial Value | Updated Value (at 6th iteration) |
|---|---|---|---|
| M16 | Density of concrete | 2500 kg/m$^3$ | 2475-2525 kg/m$^3$ |
| M15 | Elastic modulus of stay cables | 195 GPa | 195.5 GPa |
| G8 | Cross sectional area of girder | 23 m$^2$ | 17.5 m$^2$ |
| M13 | Concrete strength of towers and piers | 40 MPa | 32-40 MPa |
| G9 | Moment of inertia of girder | 21 m$^4$ | 27.25 m$^4$ |
| M14 | Concrete strength of girder | 55 MPa | 60 MPa |
| NS12 | Non-structural girder mass per unit length | 15000 kg/m | 14203 kg/m |

# 4. Model Updating – Uses of updated model

- Baseline reference model
- Assisting in long-term monitoring of the bridge in conjunction with the SHM system
  - Assisting in damage detection and damage simulation studies
  - Assessing cable tension forces
  - Identifying global behaviour with a limited number of on-structure sensors

# 4. Model Updating – Findings and challenges

- SHM data variability throughout the year
  - Varying environmental conditions = a range of values
  - FE model output = single value
  - Acknowledging and incorporating data variability

- The measurement weighting matrix
  - Reflects certainty/uncertainty in data
  - Prioritizes modes for updating

- Matching and identifying measured and FE model modes
  - Measurement uncertainty
  - FE model uncertainty i.e. modelling assumptions
  - More complex FE model $\neq$ more accuracy

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# Thank you.