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Condition Assessment based on Modal Feature Correlation

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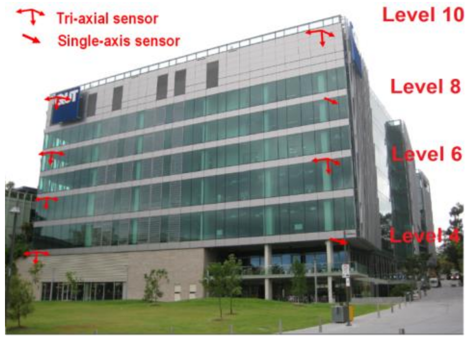
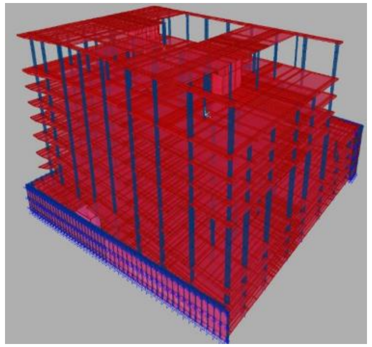
Outline

- ❑ Introduction
- ❑ Correlation-based Damage Identification for structures with high uncertainties
- ❑ Correlation-based Damage Identification for mass-varied structures
- ❑ Summary

Introduction





❑ Vibration-based Damage Identification

- Use the change in modal parameters (e.g., natural frequency, mode shape, modal flexibility, modal curvature, modal strain energy, etc.) to inversely estimate the change in structural properties (e.g., stiffness reduction).
- Cost effective and practical when used in conjunction with ambient vibration measurement (without disruption for normal operation of structures) .
- Enable real-time and continuous assessment.

Petrochemical Pipe Rack
(copyright by hardhatengineer.com)

❑ Practical Challenges:

- Uncertainties (measurement noise, modelling errors)
- Change in non-structural factors (e.g., mass variation) between undamaged and damaged states

Bridge at different traffic condition
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Damage Identification based on Modal Feature Change Correlation

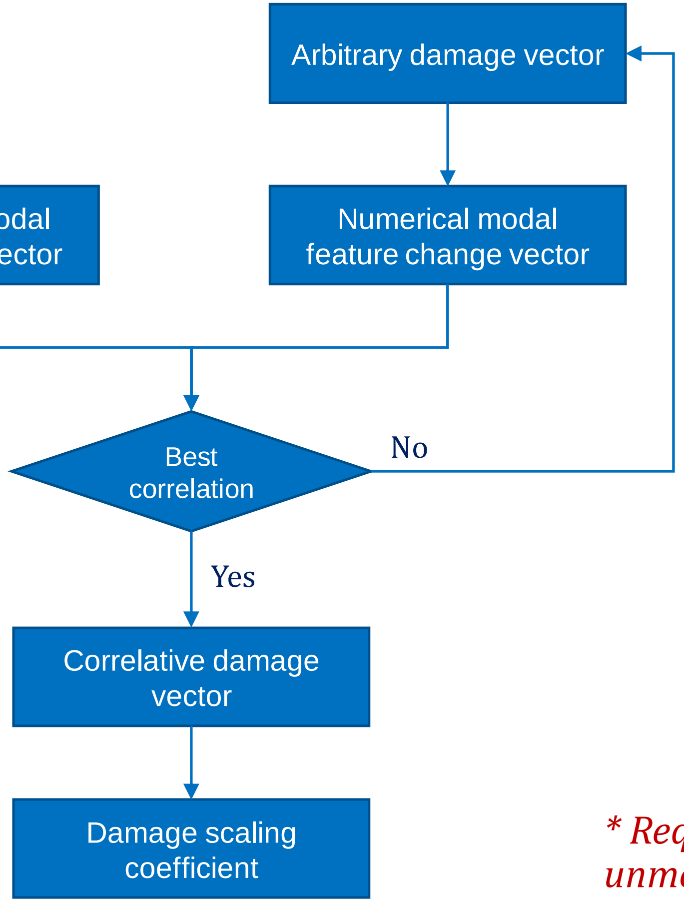
Maximize $$\mathrm{MDLAC}^{Z}(\boldsymbol{\delta}\mathbf{D}) = \frac{\left|\Delta\mathbf{Z}^{\mathrm{T}}.\boldsymbol{\delta}\mathbf{Z}(\boldsymbol{\delta}\mathbf{D})\right|^{2}}{\left(\Delta\mathbf{Z}^{\mathrm{T}}.\Delta\mathbf{Z}\right).\left(\Delta\mathbf{Z}(\boldsymbol{\delta}\mathbf{D})^{\mathrm{T}}.\Delta\mathbf{Z}(\boldsymbol{\delta}\mathbf{D})\right)}$$

Experimental modal feature change

Change in Natural Frequency $\Delta\mathbf{f}$

Change in Mode Shape $\Delta\boldsymbol{\Phi}$

Change in Modal Strain Energy $\Delta\mathbf{U}$



Numerical modal feature change

$$\frac{\partial f_i}{\partial D_k} = \frac{1}{8 f_i \pi^2} \cdot \frac{\boldsymbol{\Phi}_i^T \mathbf{K}_k \boldsymbol{\Phi}_i}{\boldsymbol{\Phi}_i^T \mathbf{M} \boldsymbol{\Phi}_i}$$

$$\frac{\partial \boldsymbol{\Phi}_i}{\partial D_k} = -\sum_{r=1}^{n} \frac{\boldsymbol{\Phi}_r^T \mathbf{K}_k \boldsymbol{\Phi}_i}{\lambda_r - \lambda_i} \boldsymbol{\Phi}_r \, (r \neq i)$$

$$\frac{\partial U_{ij}}{\partial D_k} = -2\sum_{r=1}^{n} \boldsymbol{\Phi}_i^{\mathrm{T}} \mathbf{K}_j \frac{\boldsymbol{\Phi}_r^{\mathrm{T}} \mathbf{K}_k \boldsymbol{\Phi}_i}{\lambda_r - \lambda_i} \boldsymbol{\Phi}_r \qquad (r \neq i)$$

** Requires numerical modal information for unmeasured/unmeasurable high order modes*

Approximate method using only experimental modes: MSEEC Correlation

- According to Kim and Stubbs (1995), assume $F_{ij} = F^*_{ij}$, change of *elemental modal strain energy-eigenvalue* (MSEE) ratio:

$$\mathrm{d}\,MSEE_{ij} = -\frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{K}_j\mathbf{\Phi}_i}{\lambda_i}\mathrm{d}\,D_j = -S_{ij}^{MSEE}dD_j \qquad (1)$$

- Considering the fact $\sum_{j=1}^{n} F_{ij} = \sum_{j=1}^{n} F_{ij}^* = 1$, change of *total MSEE* ratio:

$$\mathrm{d}\,TMSEE_i = -\sum_{j=1}^{n}\frac{\mathbf{\Phi}_i^{\mathrm{T}}\mathbf{K}_j\mathbf{\Phi}_i}{\lambda_i}\mathrm{d}\,D_j = -\sum_{j=1}^{n} S_{ij}^{MSEE}dD_j \qquad (2)$$

- **Identification of damage location & correlative damage vector**

Maximize $$\mathrm{MDLAC}^{\mathrm{MSEE}}(\delta\mathbf{D}) = \frac{\left|\Delta\mathbf{MSEE}^{\mathrm{T}}.\delta\mathbf{MSEE}\right|^2}{\left(\Delta\mathbf{MSEE}^{\mathrm{T}}.\Delta\mathbf{MSEE}\right).\left(\delta\mathbf{MSEE}^{\mathrm{T}}.\delta\mathbf{MSEE}\right)}$$

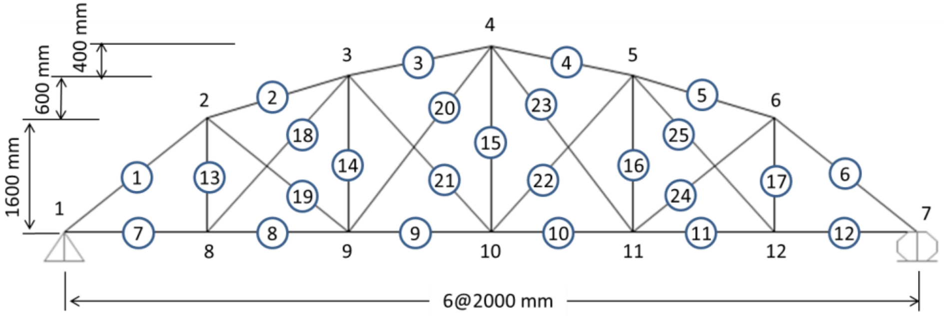
$$\Delta\mathbf{MSEE} = \begin{Bmatrix} \begin{Bmatrix}\Delta\mathbf{MSEE}_1 \\ \Delta TMSEE_1\end{Bmatrix} \\ \cdots \\ \begin{Bmatrix}\Delta\mathbf{MSEE}_i \\ \Delta TMSEE_i\end{Bmatrix} \\ \cdots \\ \begin{Bmatrix}\Delta\mathbf{MSEE}_m \\ \Delta TMSEE_m\end{Bmatrix} \end{Bmatrix}$$

Combining MSEE change and TMSEE change provides more reliable identification, due to:

✓ MSEE change is local parameter (**sensitive** to damage) but **less accurately** estimated.

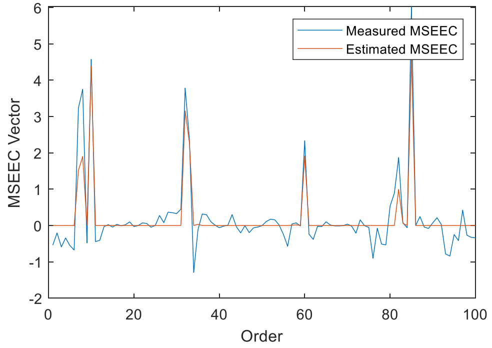
✓ TMSEE change is global parameter (**less sensitive** to damage) but **more accurately** estimated.

Noise Free: Approximate method vs Exact Method



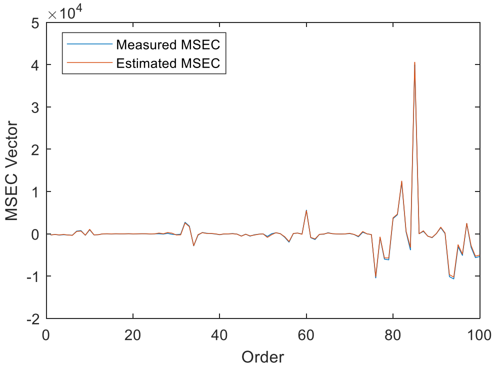
Correlation levels of modal feature changes

Approximate Method
MSEE Correlation

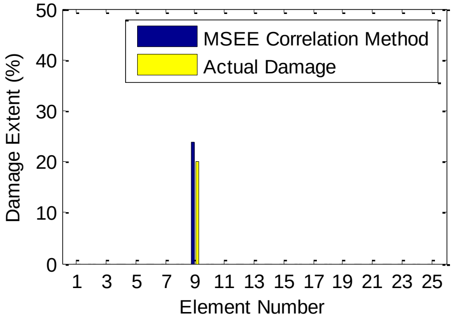
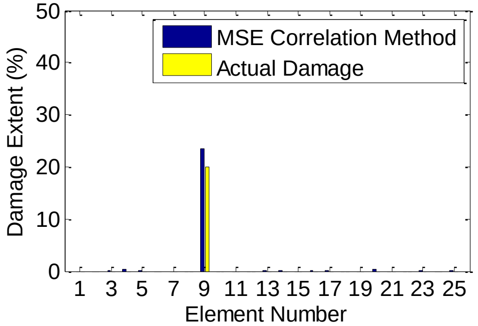
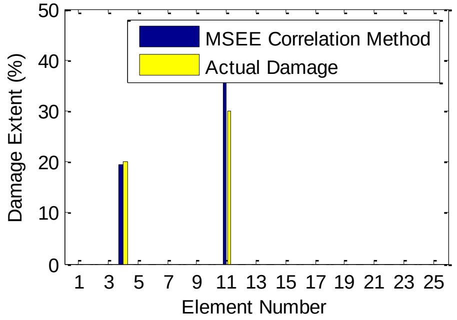
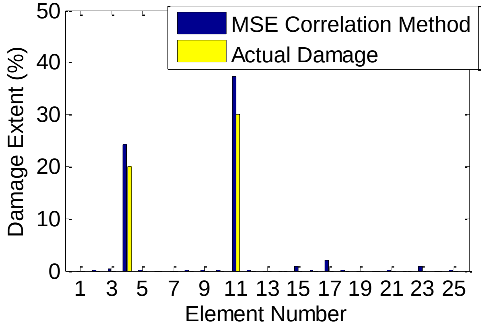
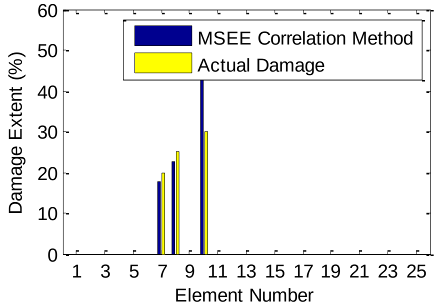
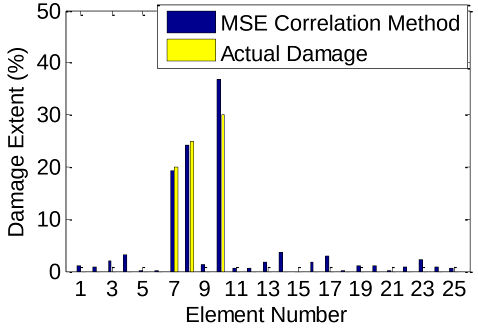


91.9% Correlation for case 3

Exact Method
MSE Correlation



99.9% Correlation for case 3

	Approximate Method MSEE Correlation	**Exact Method** MSE Correlation
Case 1 #9(20%)		
Case 2 #4(20%) #11(30%)		
Case 3 #7(20%) #8(25%) #10(30%)		

Measurement Noise: Approximate method vs Exact Method

Frequency noise = 1%, Mode shape noise = 5%

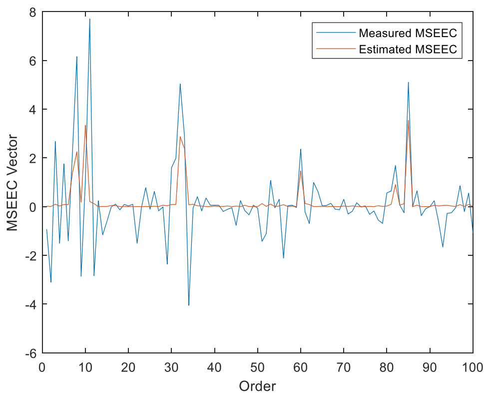
Damage quantification

Approximate (MSEE) vs Exact (MSE) Method

Damage scenario	Damaged elements	MSE Error (%)	MSEE Error (%)
Case 1	9	18.00	8.30
Case 2	4	18.34	8.90
	11	30.82	13.70
Case 3	7	6.20	3.00
	8	8.70	0.50
	10	15.00	13.60
Average error		16.18	8.00

Approximate Method

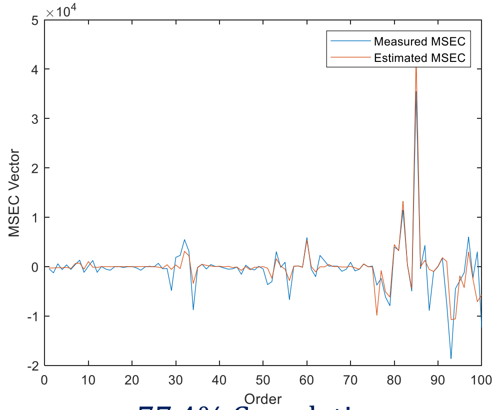
MSEE Correlation



46% Correlation for case 3

Exact Method

MSE Correlation

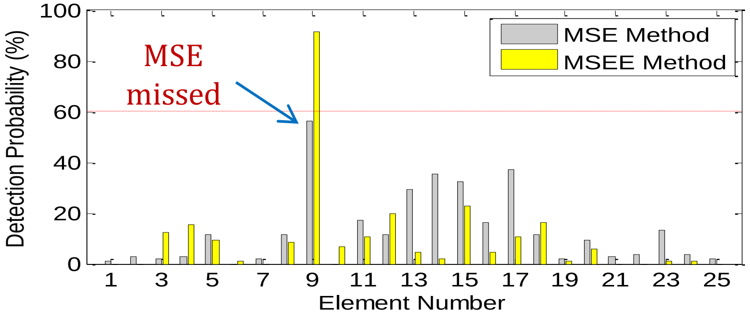


77.4% Correlation for case 3

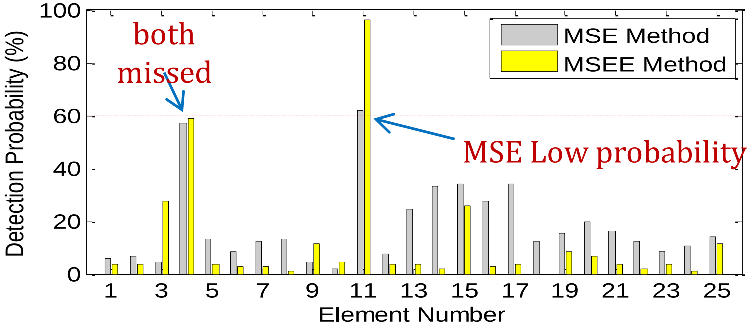
Detection Probability

Approximate (MSEE) vs Exact (MSE) Method

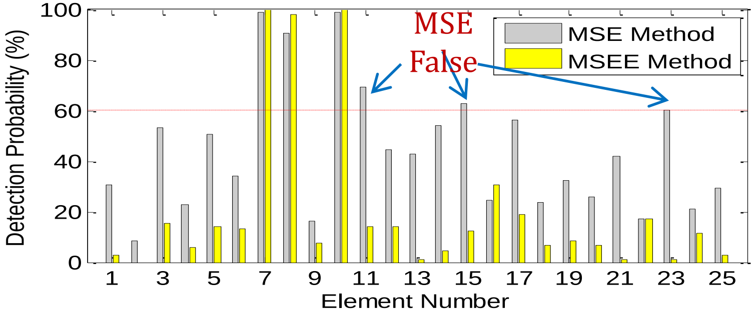
Case 1
#9(20%)



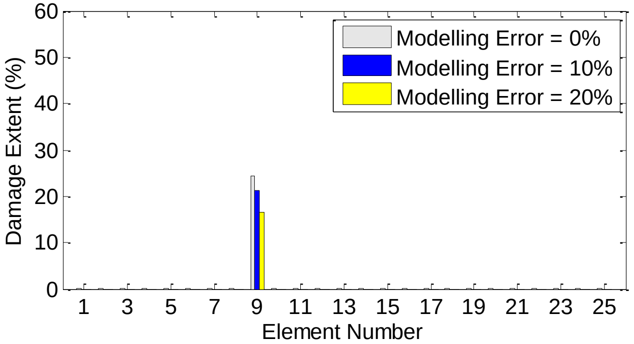
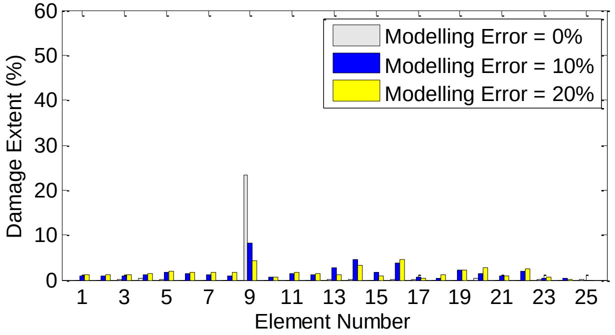
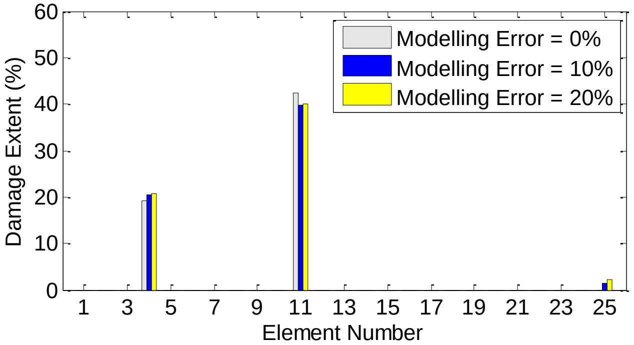
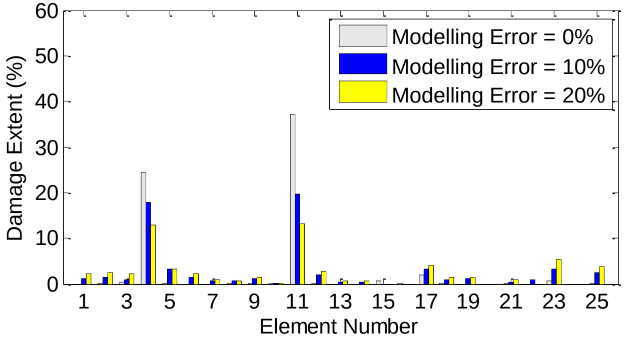
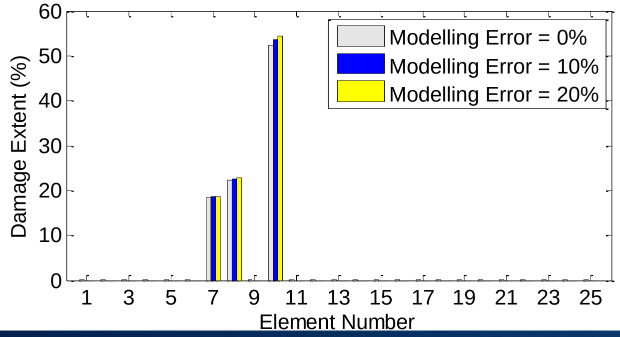
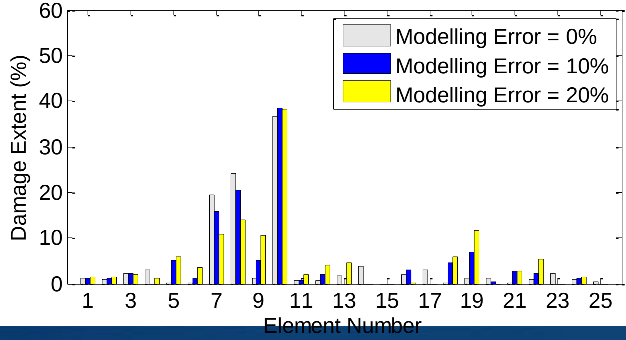
Case 2
#4(20%)
#11(30%)



Case 3
#7(20%)
#8(25%)
#10(30%)



Modelling Error: Approximate method vs Exact Method

	Approximate Method MSEE Correlation	Exact Method MSE Correlation
Case 1 #9(20%)		
Case 2 #4(20%) #11(30%)		
Case 3 #7(20%) #8(25%) #10(30%)		

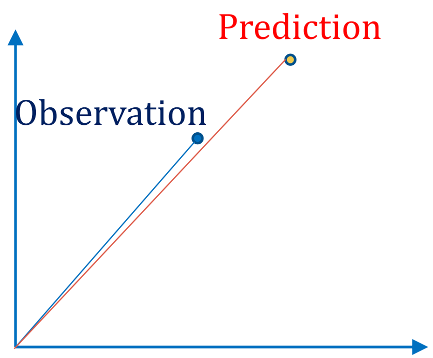
Correlation-based vs Error-based Objective Functions

Correlation-based Objective Function

Maximize $$\mathrm{MDLAC}^{Z}(\boldsymbol{\delta}\mathbf{D}) = \frac{\left|\Delta\mathbf{Z}^{T}.\boldsymbol{\delta}\mathbf{Z}(\boldsymbol{\delta}\mathbf{D})\right|^{2}}{\left(\Delta\mathbf{Z}^{T}.\Delta\mathbf{Z}\right).\left(\Delta\mathbf{Z}(\boldsymbol{\delta}\mathbf{D})^{T}.\Delta\mathbf{Z}(\boldsymbol{\delta}\mathbf{D})\right)}$$

Requires two steps:

- Step 1: Correlative damage vector
- Step 2: Damage scaling coefficient

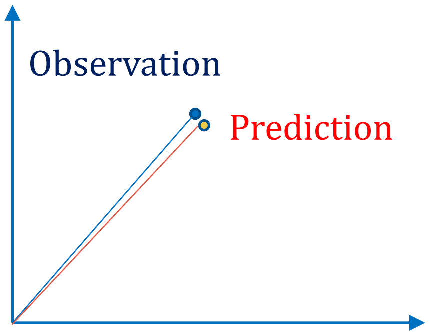


MDLAC minimizes only the angle between observation and prediction

Error-based Objective Function

Minimize $$\mathrm{RMSD}^{Z}(\boldsymbol{\delta}\mathbf{D}) = \frac{\left|\Delta\mathbf{Z} - \boldsymbol{\delta}\mathbf{Z}(\boldsymbol{\delta}\mathbf{D})\right|^{2}}{\left|\Delta\mathbf{Z}\right|^{2}}$$

One step only to provide final damage result

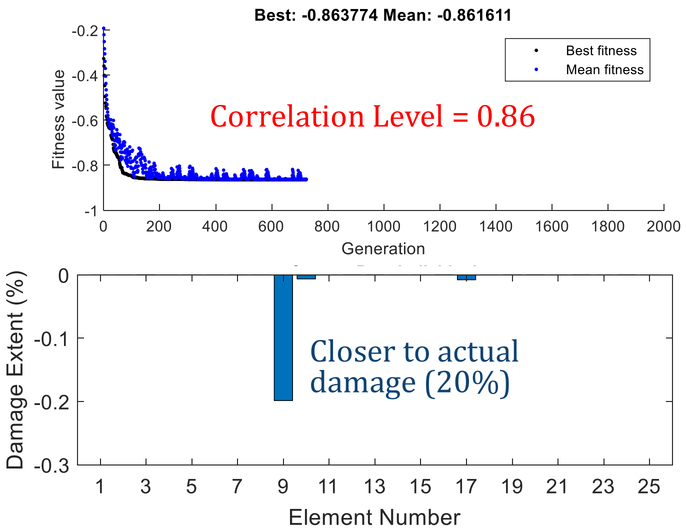


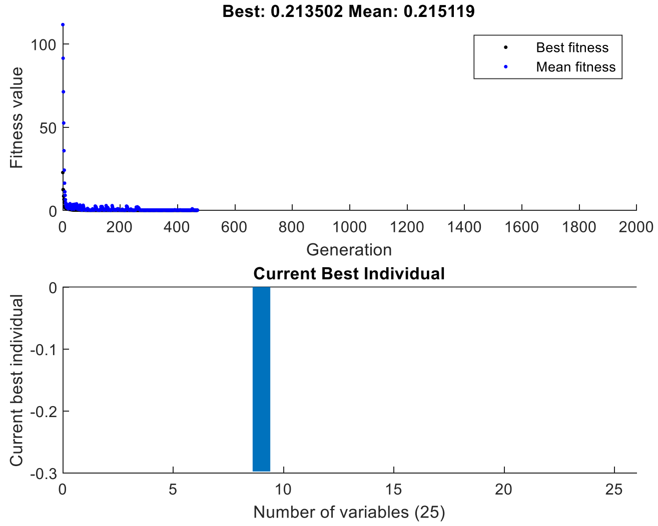
RMSD minimizes both the angle and magnitude between observation and prediction

Correlation-based Objective Function

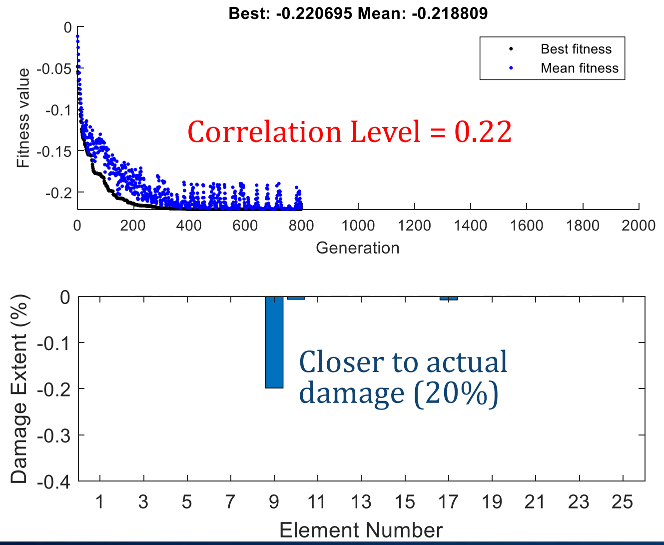
Error-based Objective Function

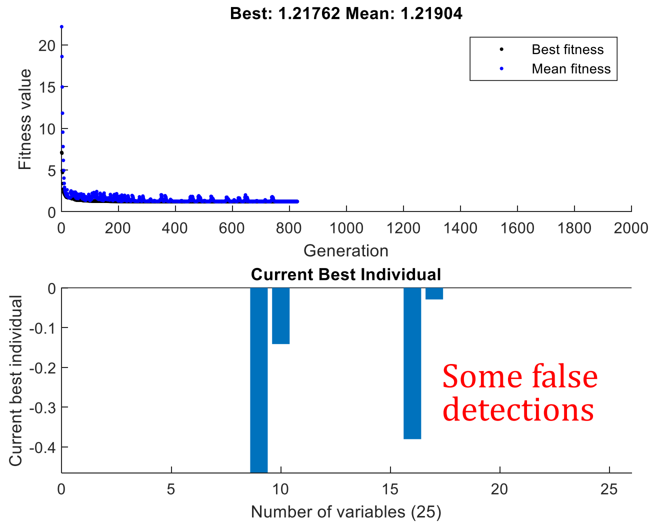
Noise Free



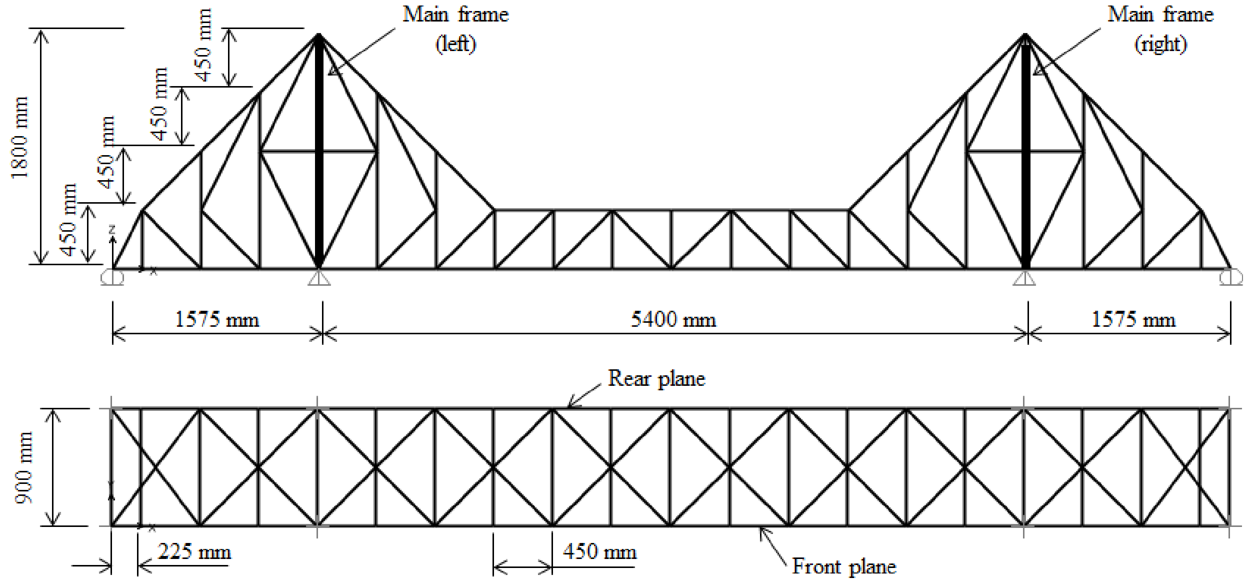


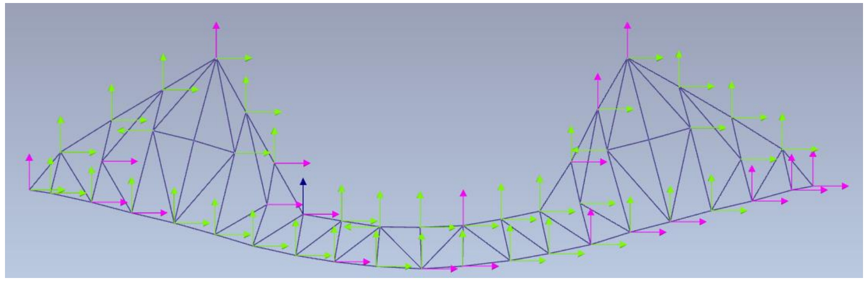
Noisy Condition



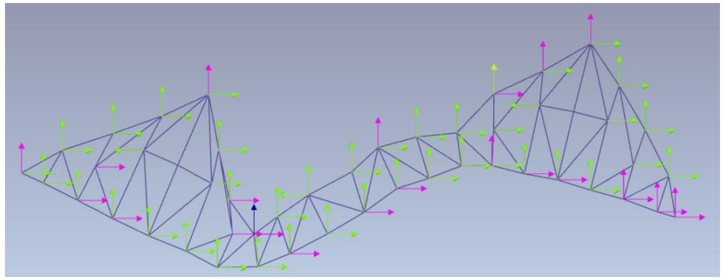


Experimental Validation

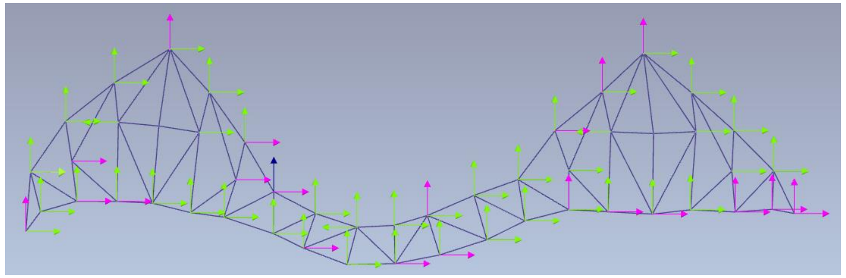




Mode 1: 15.375Hz (FEM = 15.185 Hz)
MAC = 0.653



Mode 2: 30.25Hz (FEM = 30.854 Hz)
MAC = 0.304



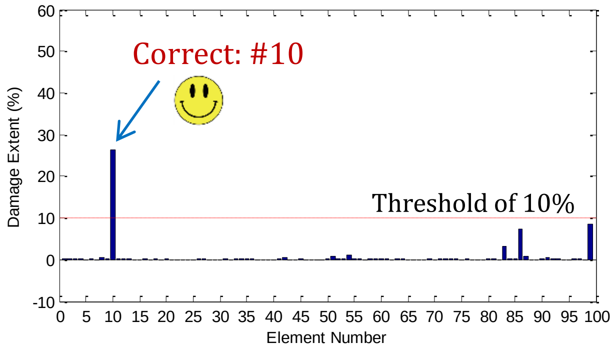
Mode 3: 58.75Hz (FEM = 59.480 Hz)
MAC = 0.598

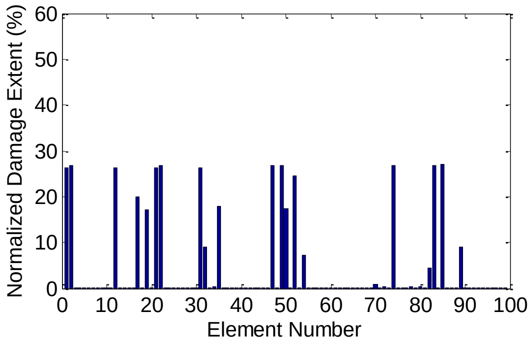
Experimental Validation: Results

MSEE Correlation

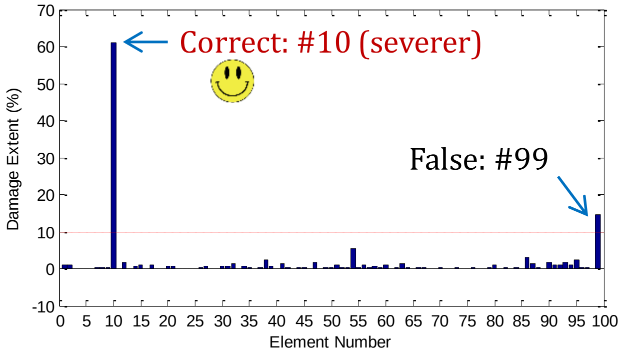
MSE Correlation

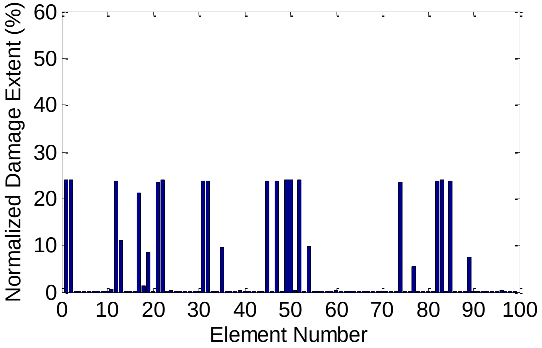
State 1-1
Damage at 10
(2 bolts loosened)



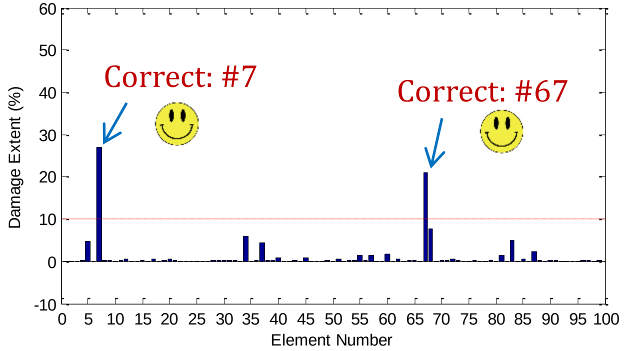


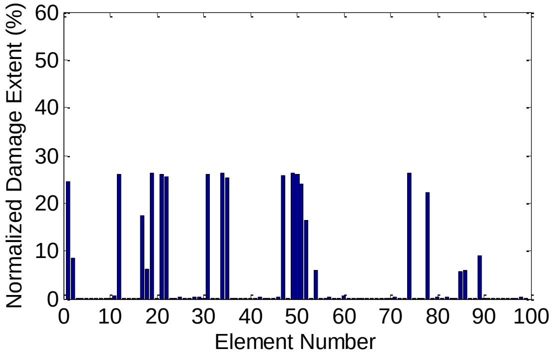
State 1-2
Damage at 10
(4 bolts loosened)





State 2-1
Damage at 7 & 67
(4 bolts loosened)





Damage Detection for mass-varied structures: MKE-based correlation

Maximize $$MDLAC^{MKE} = \frac{\left|\{R^{MKE}\}^T\{\Re^{MKE}\}\right|^2}{(\{R^{MKE}\}^T\{R^{MKE}\})\left(\{\Re^{MKE}\}^T\{\Re^{MKE}\}\right)}$$

Numerical MKE-based damage indicator

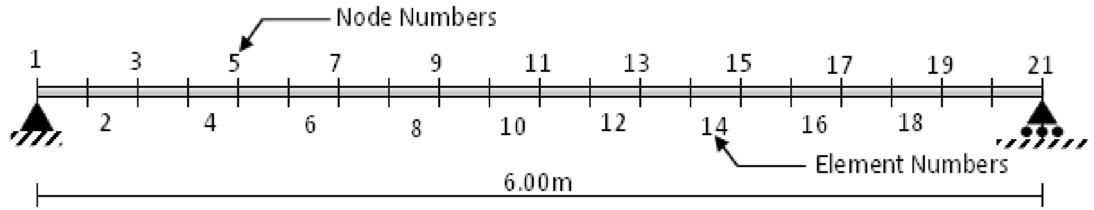
$$\Re_{i,j}^{MKE} = \sum_{p=1}^{L} S_{i,j,p}^{MKE} . \alpha_p$$

where $$S_{i,j,p}^{MKE} = -2\{\phi_i\}^T[M]_j\left\{\sum_{r=1}^{n}\frac{\{\phi_r\}^T[K]_p\{\phi_i\}}{\lambda_r-\lambda_i}\{\phi_r\}\right\}$$

Corresponding measured MKE-based damage indicator

$$R_{i,j}^{MKE} = \frac{\lambda_i^d}{\lambda_i}\{\phi_i^d\}^T[M^d]_j\{\phi_i^d\} - \{\phi_i\}^T[M]_j\{\phi_i\} - \{\phi_i\}^T[\Delta M]_j\{\phi_i\} - \frac{\Delta\lambda_i}{\lambda_i}\{\phi_i\}^T[M]_j\{\phi_i\}$$
$$-\sum_{p=1}^{L} 2\lambda_i\{\phi_i\}^T[M]_j\left\{\sum_{r=1}^{n}\frac{\{\phi_r\}^T[\Delta M]_p\{\phi_i\}}{\lambda_r-\lambda_i}\{\phi_r\}\right\} + \sum_{p=1}^{L}\{\phi_i\}^T[M]_j\left\{\left(\{\phi_i\}^T[\Delta M]_p\{\phi_i\}\right)\{\phi_i\}\right\}$$

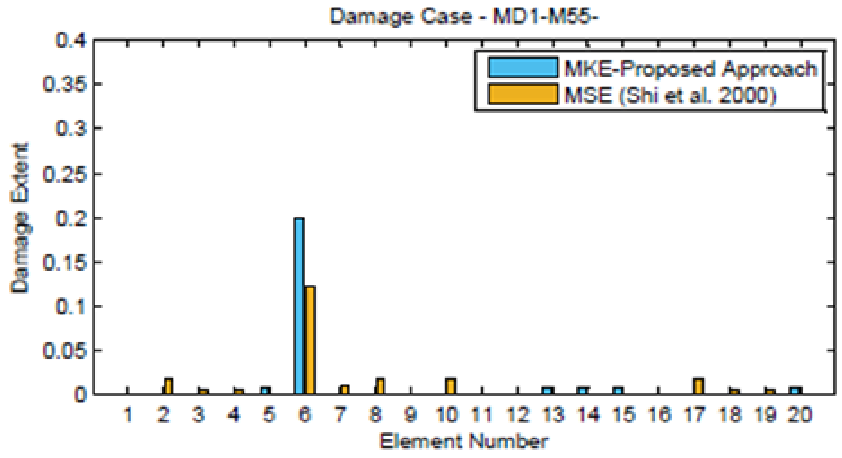
Numerical Validation



Case	Damage	Mass variation
Single Damage (MD1_M55-)	#6 (20%)	#6 (-5%); #14 (-5%)
Single Damage (MD1_M55+)	#6 (20%)	#6 (+5%), #14 (+5%)
Multiple Damage (MD2_M55-)	#6 (10%); #11 (15%)	#6 (-5%); #14 (-5%)
Multiple Damage (MD2_M55+)	#6 (10%); #11 (15%)	#6 (+5%); #14 (+5%)

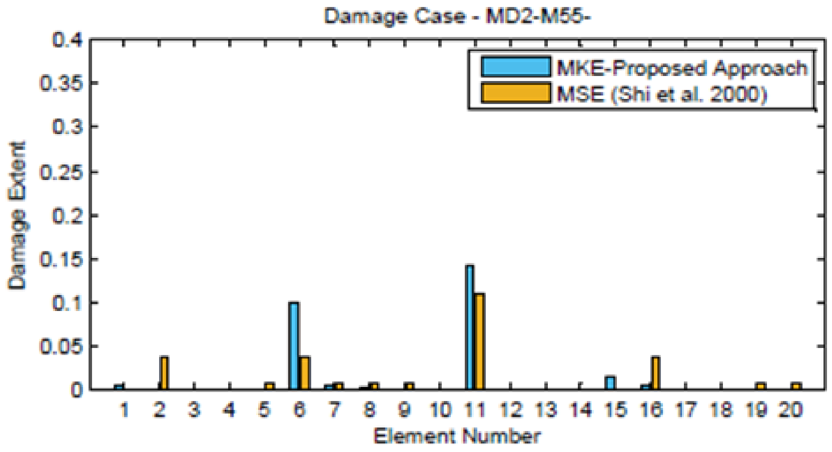
MD1-M55-

Damage: #6 (20%), mass: -5% at #6&#14



MD2-M55-

Damage: #6 (10%), #11 (15%), mass: -5% at #6&#14



Improved MKE-based correlation for large mass variation

Maximize $$MDLAC^{MKE} = \frac{\left|\{R^{MKE}\}^T\{\Re^{MKE}\}\right|^2}{(\{R^{MKE}\}^T\{R^{MKE}\})\left(\{\Re^{MKE}\}^T\{\Re^{MKE}\}\right)}$$

Original Numerical MKE-based damage indicator

$$\Re_{i,j}^{MKE} = \sum_{p=1}^{L} S_{i,j,p}^{MKE}.\alpha_p$$

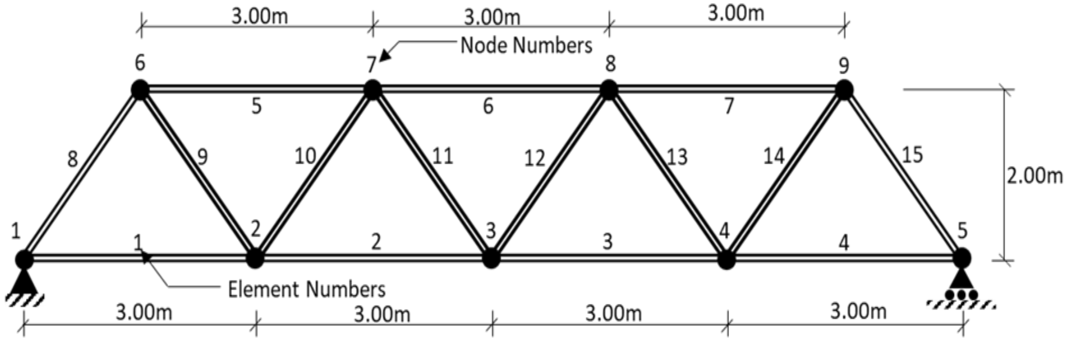
where $$S_{i,j,p}^{MKE} = -2\{\phi_i\}^T[M]_j\left\{\sum_{r=1}^{n}\frac{\{\phi_r\}^T[K]_p\{\phi_i\}}{\lambda_r-\lambda_i}\{\phi_r\}\right\}$$

Improved numerical MKE-based damage indicator

$$\Re_{i,j}^{MKE} = \sum_{p=1}^{L} S_{i,j,p}^{MKE}.\alpha_p$$

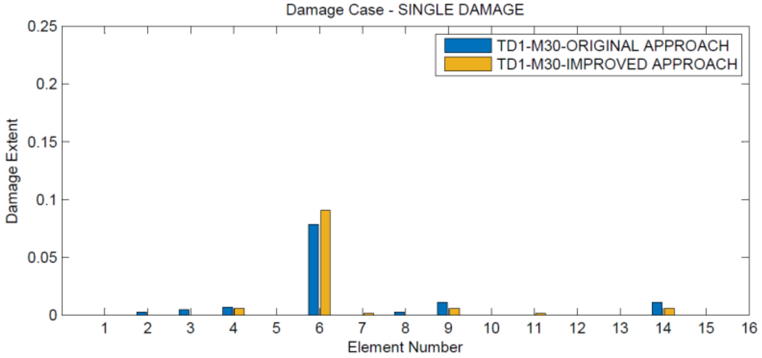
where $$S_{(imp)}{}_{i,j,p}^{MKE} = -2\{\phi_i\}^T[M]_j^d\left\{\sum_{r=1}^{n}\frac{\{\phi_r\}^T[K]_p\{\phi_i\}}{\lambda_r-\lambda_i-\lambda_i\{\phi_r\}^T[\Delta M]\{\phi_r\}}\{\phi_r\}\right\}$$

Identification of truss damage with large mass variation

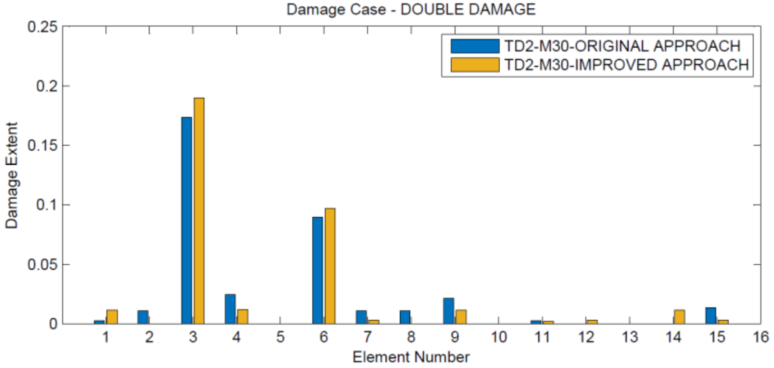


Damage Case	Damage		Mass Variation	
	Element No.	Damage Extent	Element No.	Amount
Single Damage (TD1-M30)	6	10%	6	+30%
Double Damage (TD2-M30)	6	10%	6	+30%
	3	20%	-	-
Multiple Damage (TD3-M30)	6	10%	6	+30%
	9	15%	-	-
	3	20%	-	-

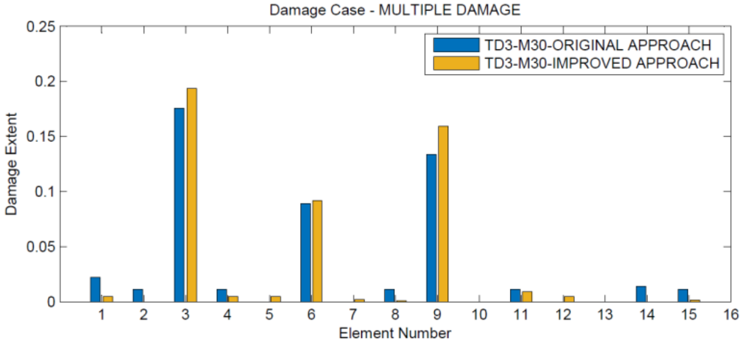
TD1-M30



TD2-M30



TD3-M30



Summary

- Approximate damage indicator using experimental modes (e.g., MSEEC) is more robust than exact damage indicators (e.g., MSEC) for structures with high uncertainties (noise, modelling error)
- Correlation-based objective function performs better than error-based objective function for weak relationship between input and output (e.g., high noise)
- The proposed MKE-based correlation approach enhances damage results for mass-varied structures

Future Study

- Adopt MSEE and machine learning/deep learning for condition assessment.
- MKE-based method for damage identification considering unknown damage-induced mass loss.

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