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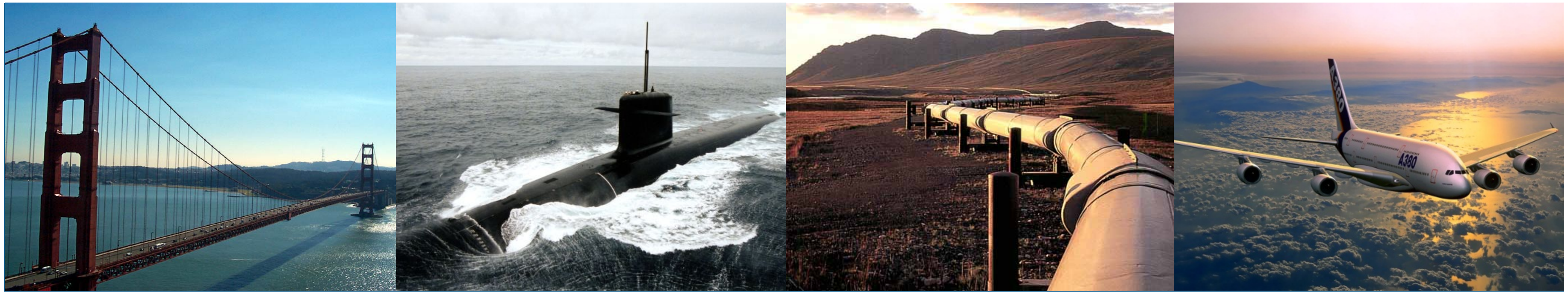


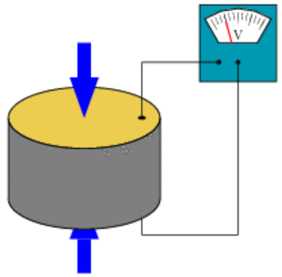
Australian Network of Structural Health Monitoring (ANSHM) 2013 Annual Workshop
19th November 2013, Melbourne, Australia

Guided Wave based Quantitative Identification of Damage in Beams Using a Bayesian Approach

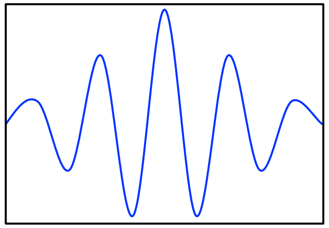
Dr. (Alex) Ching-Tai Ng

Introduction and Background

- **Guided Wave**
  - Sensitive to small and different types of damages
  - Long travel distance

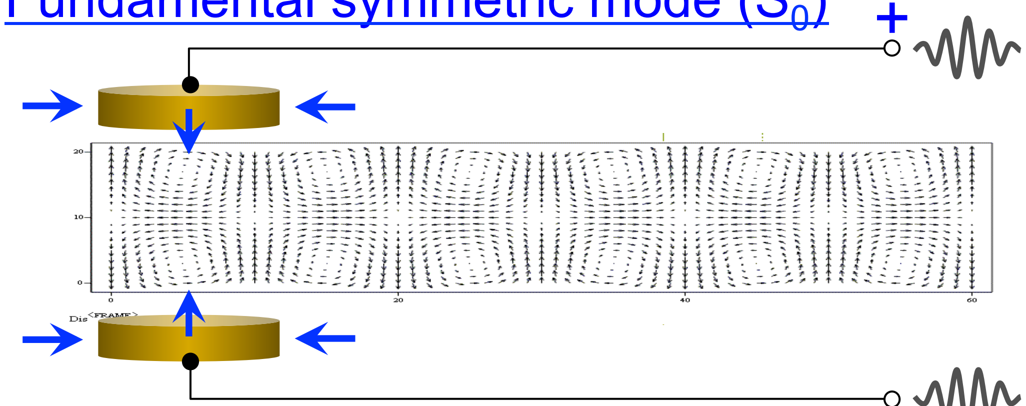


Piezoceramic transducer

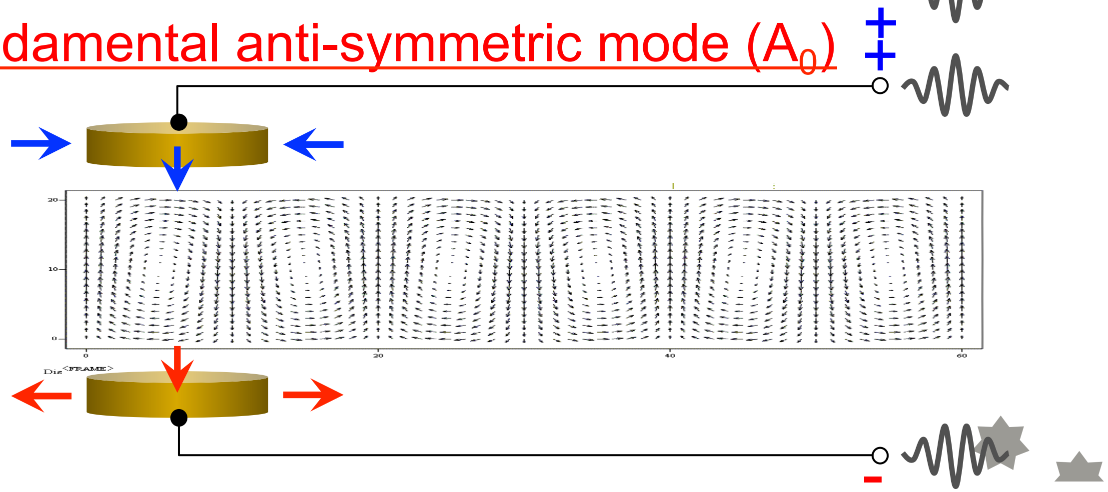


Hanning windowed sinusoidal tone burst pulse

Fundamental symmetric mode (S_0)

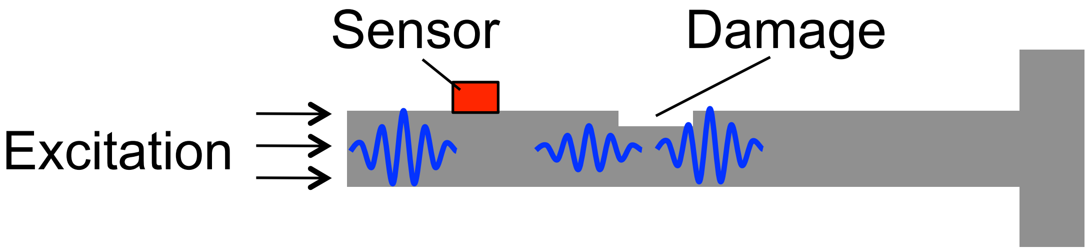


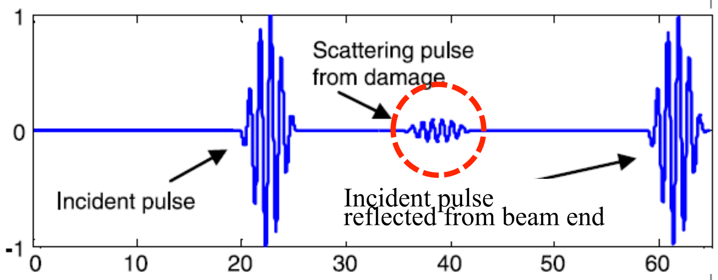
Fundamental anti-symmetric mode (A_0)



Giurgiutiu & Bao (2004) Struct. Health. Monitor. Animation from [www.me.sc.edu/Research/lamss/]

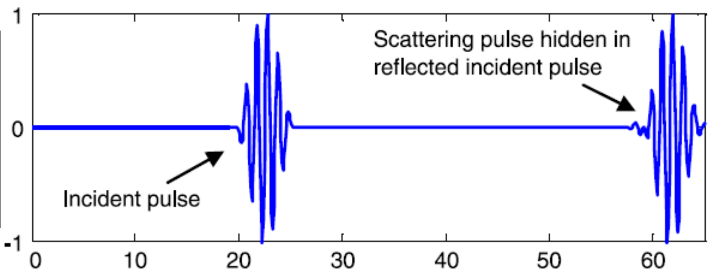
Introduction and Background





Challenges:

- Requirement of baseline data



- Temperature variation & effect of external loading conditions
- Difficult to achieve quantitative identification of damages

Objectives

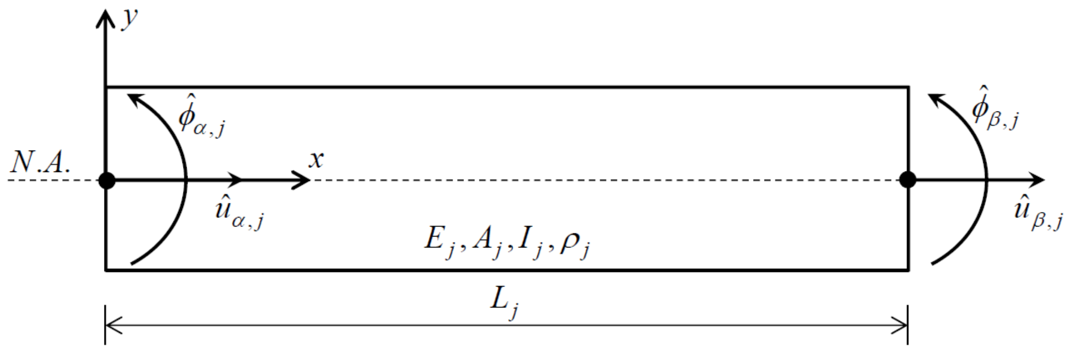
- To quantitatively identify the location and size of the damage
- To improve the computational efficiency of the proposed damage identification method using frequency domain spectral finite element simulation
- To quantify the uncertainties associated with the damage identification results using a Bayesian approach
- To provide an experimental verification of the proposed method

Frequency-domain Spectral Finite Element Method

- Mindlin-Herrmann theory
  - Describes the longitudinal wave using two coupled partial differential equations**

$$\left(2\mu_j+\lambda_j\right)A_j\frac{\partial^2\overline{u}_j}{\partial x^2}+\lambda_jA_j\frac{\partial\overline{\phi}_j}{\partial x}=\rho_jA_j\frac{\partial^2\overline{u}_j}{\partial t^2}$$

$$\mu_jI_jS_1\frac{\partial^2\overline{\phi}_j}{\partial x^2}-\left(2\mu_j+\lambda_j\right)A_j\overline{\phi}_j-\lambda_jA_j\frac{\partial\overline{u}_j}{\partial x}=\rho_jI_jS_{2,j}\frac{\partial^2\overline{\phi}_j}{\partial t^2}$$



Spectral finite element

  - Each element has 2 nodes & each node has 2 DoFs
  - Account the axial displacement & lateral contraction effect

*** Krawczuk M, Grabowska J and Palacz M, J Sound Vib 2006, 295:461-478*

Frequency-domain Spectral Finite Element Method

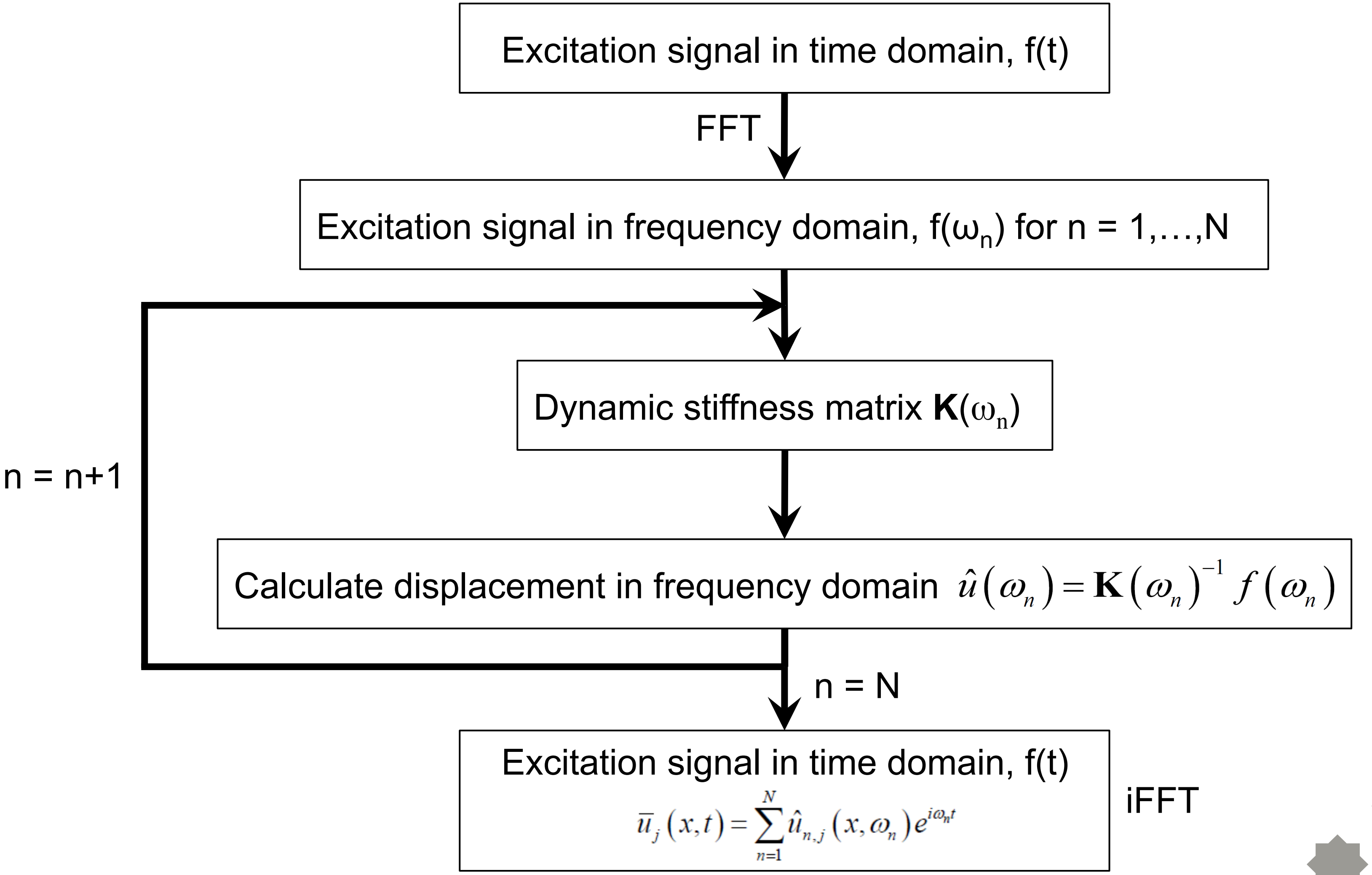
- The governing equations are reduced to two ordinary differential equations and assumes the solutions in the forms

$$\hat{u}_{n,j}(x,\omega_n) = U_j e^{-i(k_j x - \omega_n t)}, \qquad \hat{\phi}_{n,j}(x,\omega_n) = \Phi_j e^{-i(k_j x - \omega_n t)}$$

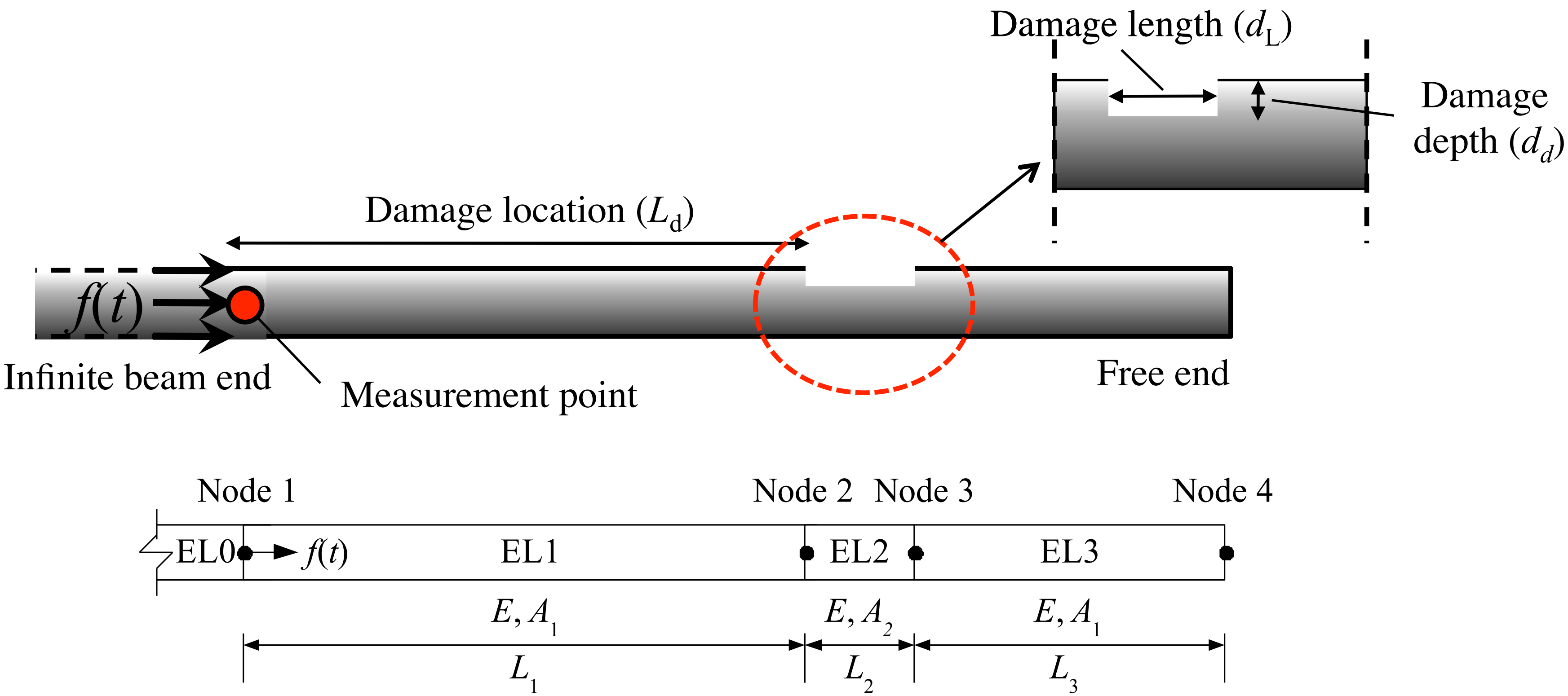
- Formulate the dynamic stiffness matrix in frequency domain (at frequency ω_n) by considering the boundary conditions

$$\mathbf{K}_{\omega_n,j} = \mathbf{T}_{2,j} T_{1,j}^{-1}$$

Frequency-domain Spectral Finite Element Method



Frequency-domain Spectral Finite Element Method



Spectral finite element model of a beam with step damage

Bayesian Approach

- Using the Bayes' theorem, the probability of the set of uncertain damage parameters (θ) with a given set of dynamic data is **:

Likelihood **Prior distribution**

$$p(\boldsymbol{\theta} \mid D, M) = c\,p(D \mid \boldsymbol{\theta}, M)\, p(\boldsymbol{\theta} \mid M)$$

where c is normalisation constant.

(Allow the inclusion of engineering judgment about the possible damage)

$$p(D \mid \boldsymbol{\theta}, M) = \left(\sqrt{2\pi}\sigma\right)^{-NN_o} e^{-\frac{NN_o}{2\sigma^2} J(\boldsymbol{\theta})}$$

where $J(\boldsymbol{\theta})$ is:

Measured signal **Simulated signal**

$$J(\boldsymbol{\theta}) = \frac{1}{NN_o} \sum_{k=1}^{N} \left\| \mathbf{q}(k) - \mathbf{S}_o \mathbf{y}(k;\boldsymbol{\theta}) \right\|^2$$

The minimisation problem is solved by Hybrid Particle Swarm Algorithm

*** Beck JL and Katafygiotis LS, J. Eng. Mech. ASCE. 1998, 124(4), 455-461*

Bayesian Approach

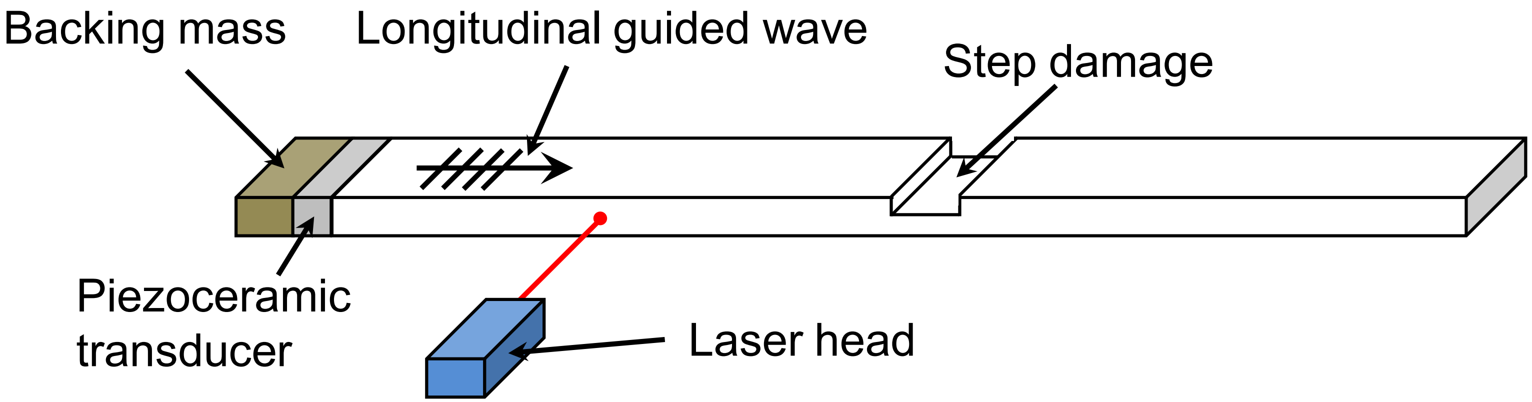
- The updated (posterior) PDF of damage parameters for given data and model class can be approximated as a weighted sum of Gaussian distributions:

$$p(\boldsymbol{\theta} \mid D) \approx \sum_{i=1}^{I} w_i N\left(\hat{\boldsymbol{\theta}}^{(i)}, \mathbf{A}^{-1}\left(\hat{\boldsymbol{\theta}}^{(i)}\right)\right)$$

- The weightings are given by:

$$w_i = \pi\left(\hat{\boldsymbol{\theta}}^{(i)}\right)\left|\mathbf{A}_N\left(\hat{\boldsymbol{\theta}}^{(i)}\right)\right|^{-\frac{1}{2}}$$

Experimental Verification



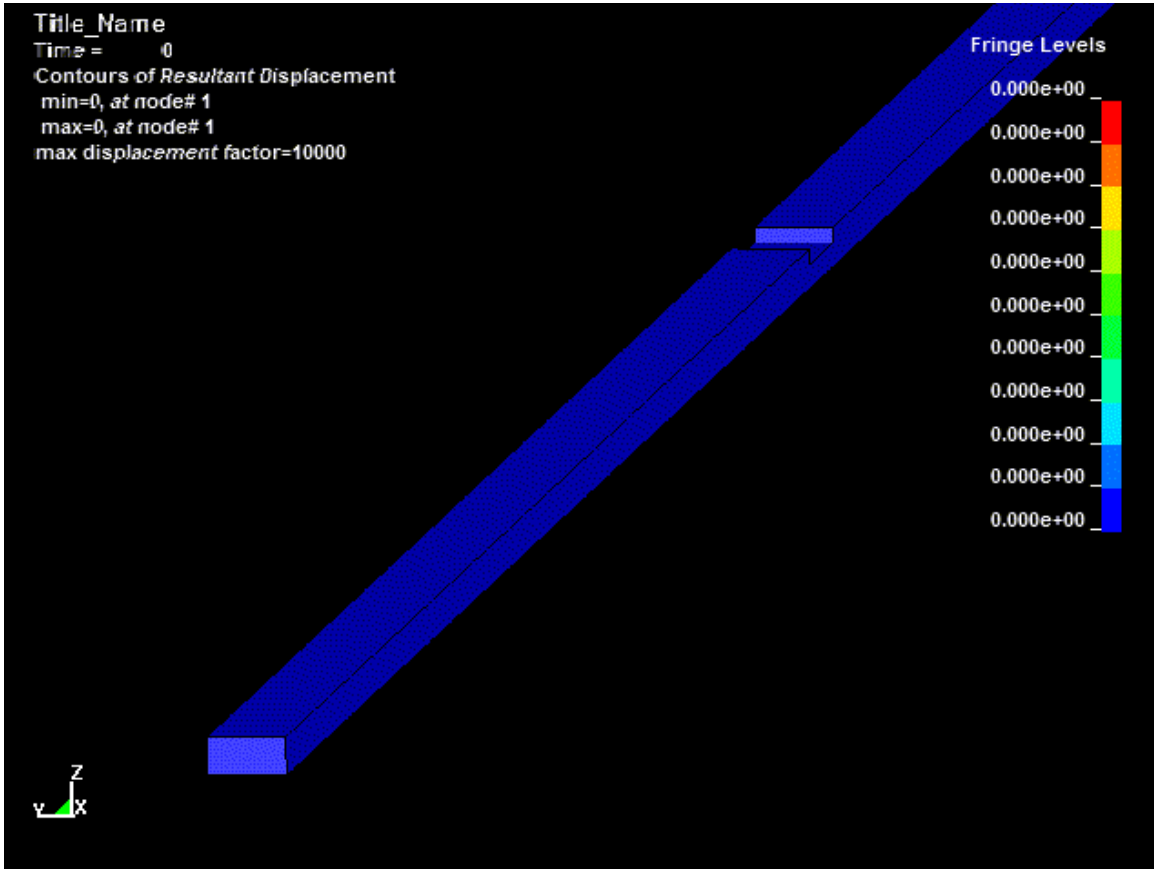
Aluminum beam
Beam cross-section: 12x6 mm^2
Beam length: 2 m
Excitation frequency: 80 kHz

Table 1. Summary of all the damage cases in the experimental case studies

Case	Damage location (L_d) (mm)	Damage length (d_L) (mm)	Damage depth (d_d) (mm)
C1	1062.50	75.00	2.00
C2	915.00	90.00	1.10

Experimental Verification

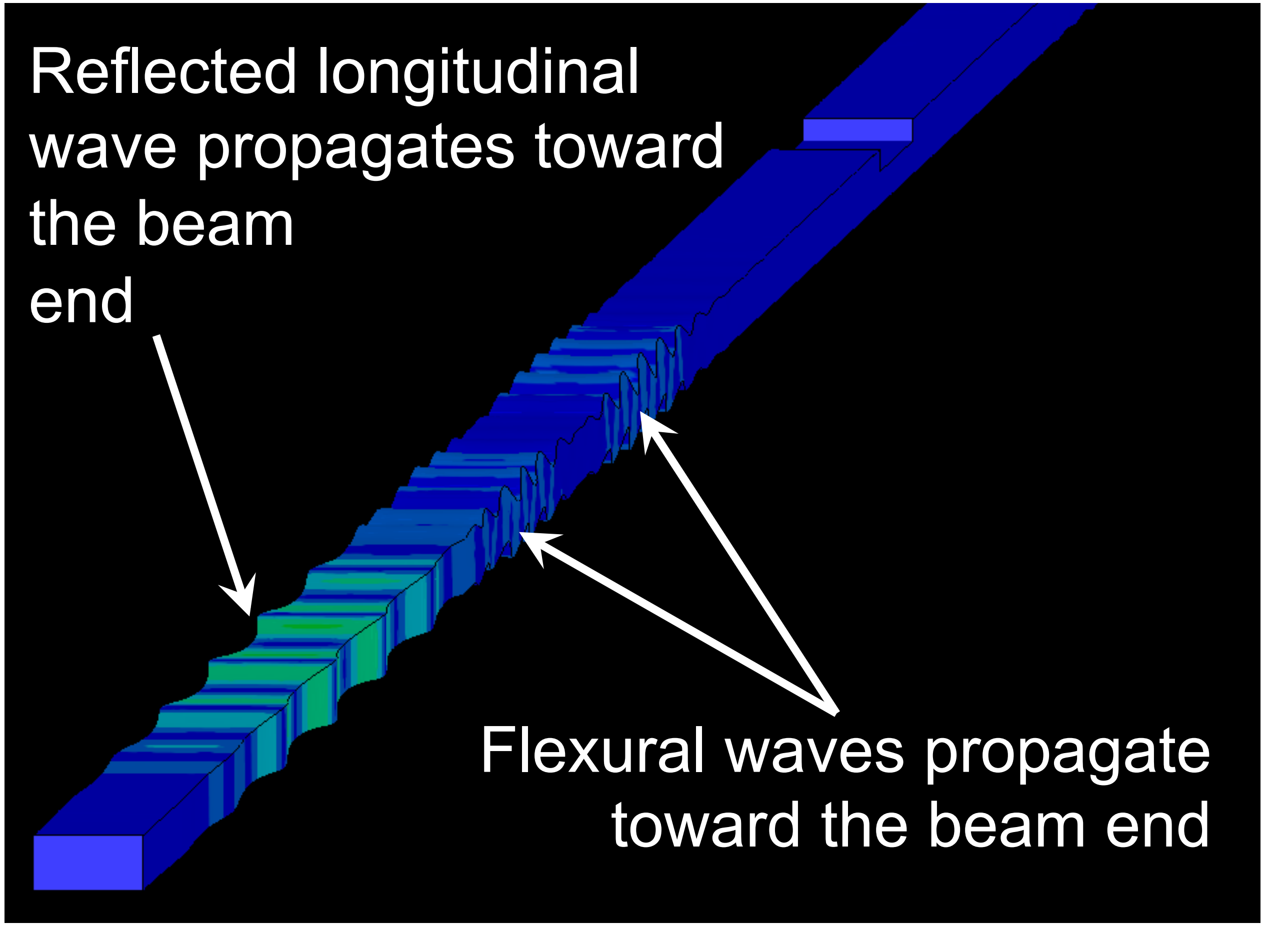
Preliminary study of measurement location using 3D finite element method (LS-DYNA)



Scaled contour plot of displacements

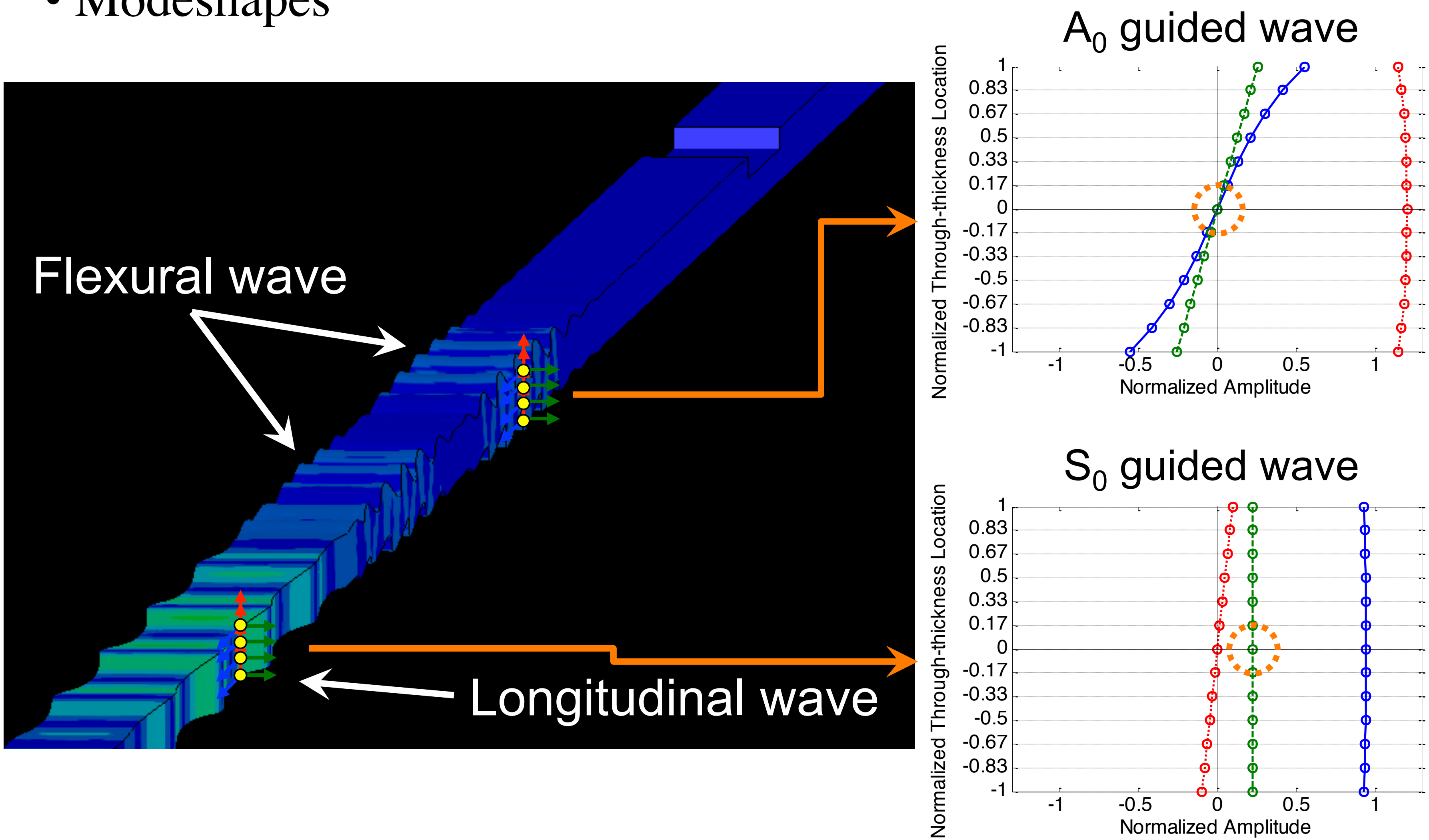
Experimental Verification

Preliminary study of measurement location using 3D finite element method (LS-DYNA)

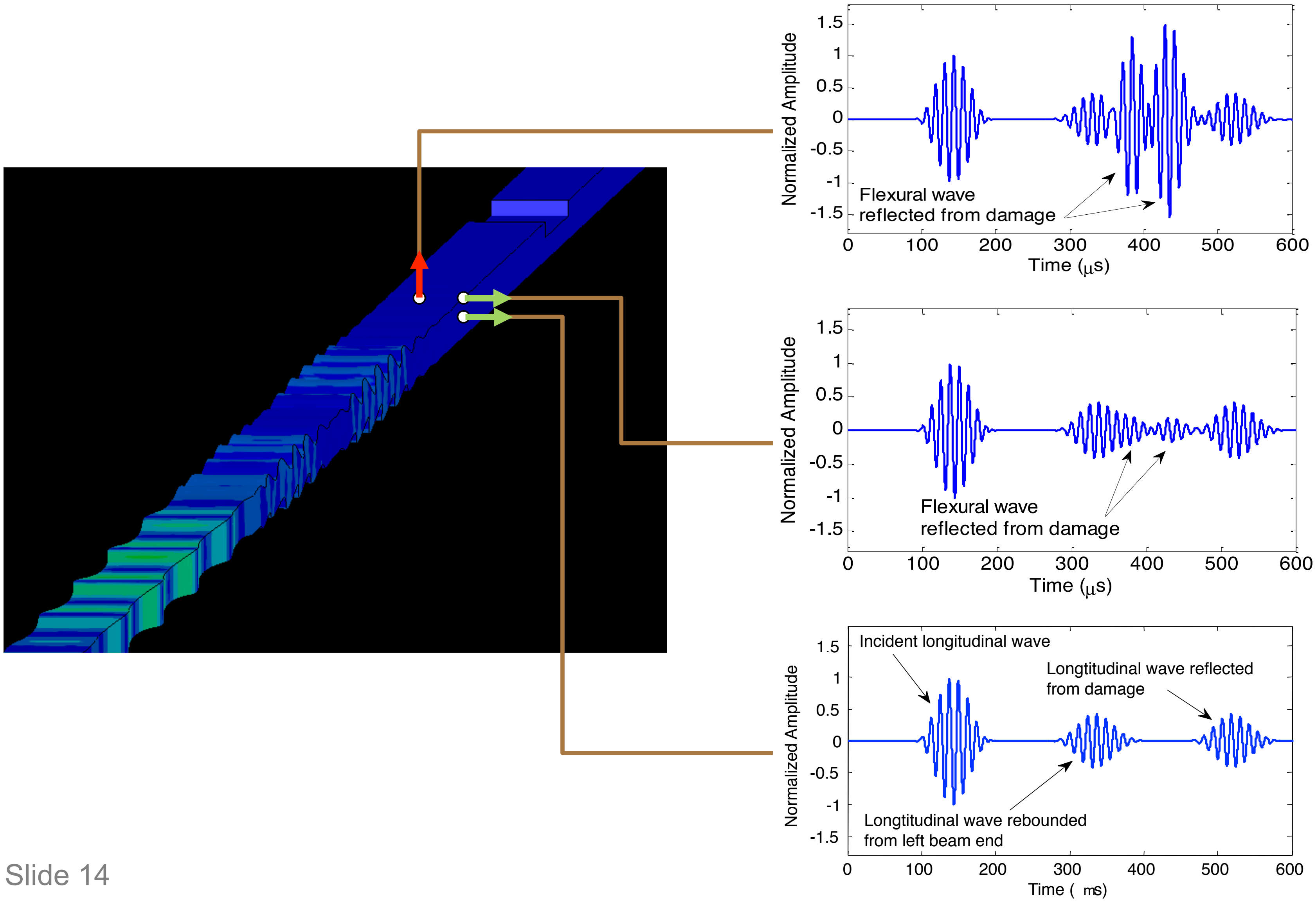


Experimental Verification

- Modeshapes



Experimental Verification



Experimental Verification

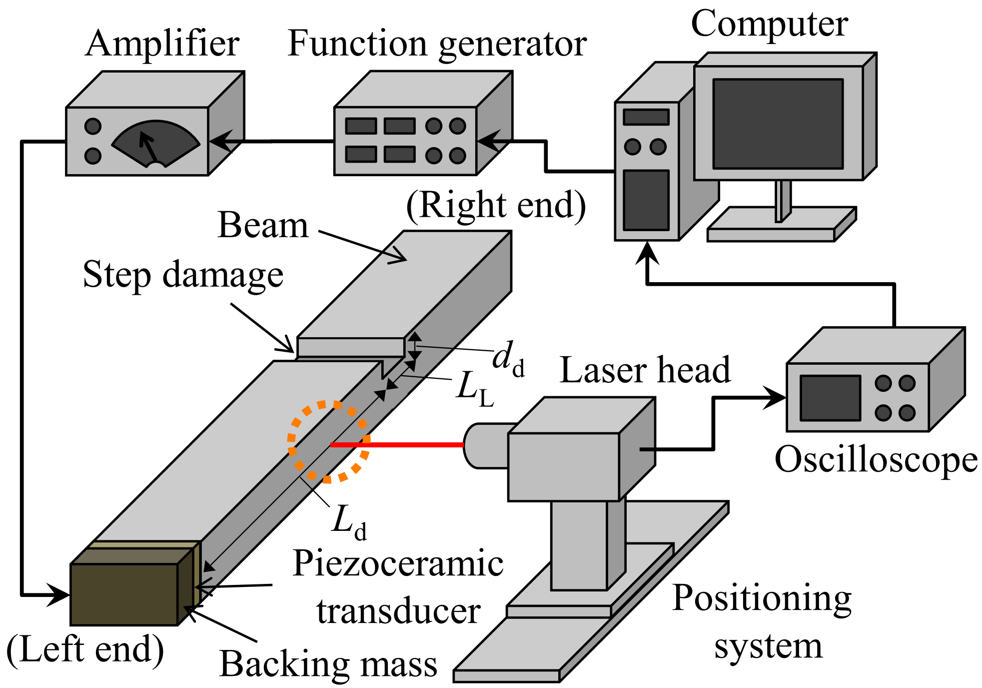


Table 2. Summary of the results in damage identification

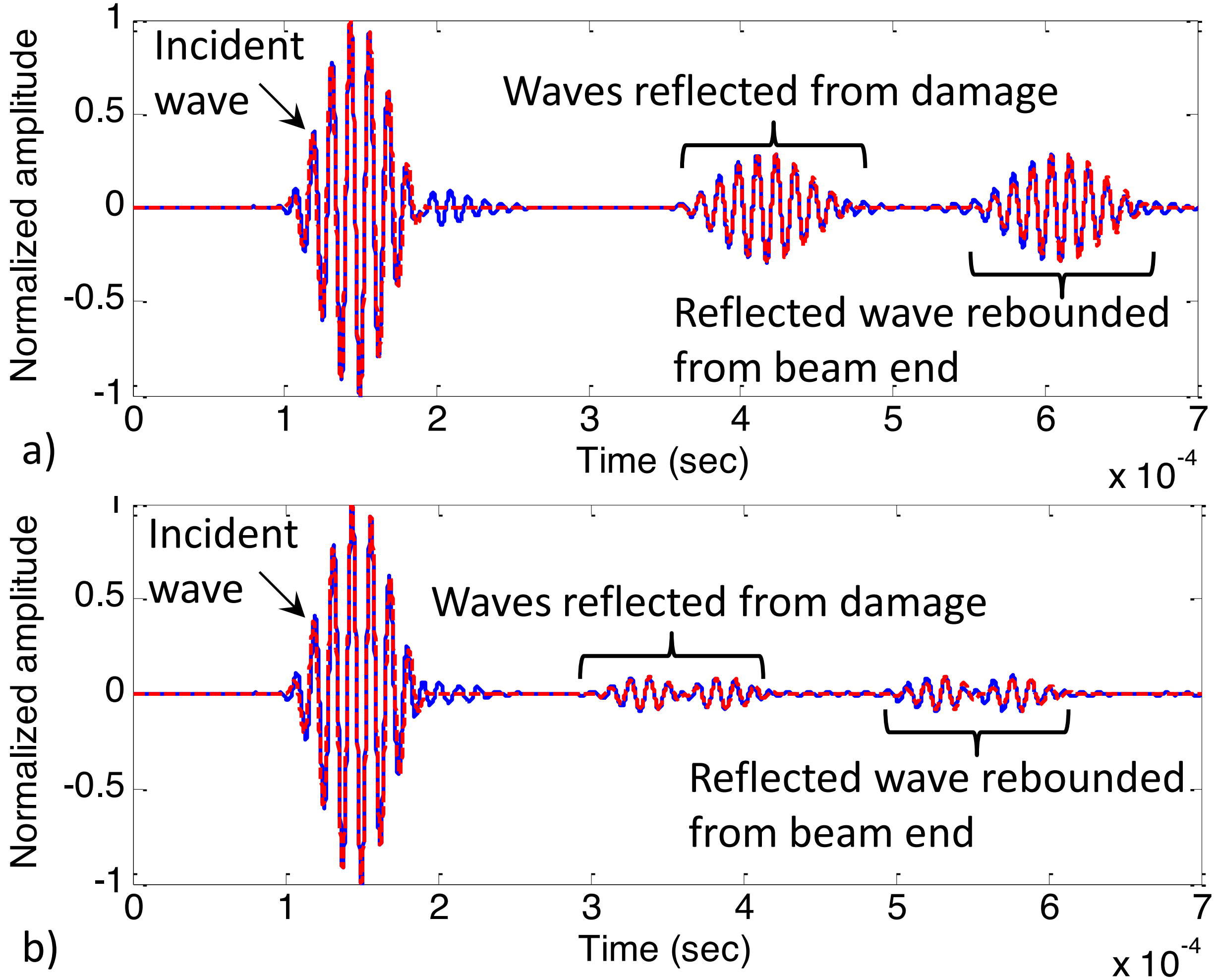
Case	Damage location (L_d) (mm)	Damage length (d_L) (mm)	Damage depth (d_d) (mm)
C1	1045.50 [1062.50]	69.71 [75.00]	2.03 [2.00]
C2	902.88 [915.00)	87.46 [90.00]	1.02 [1.10]

Note: value in square bracket is the true value of damage parameters

Experimental Verification

Case C1

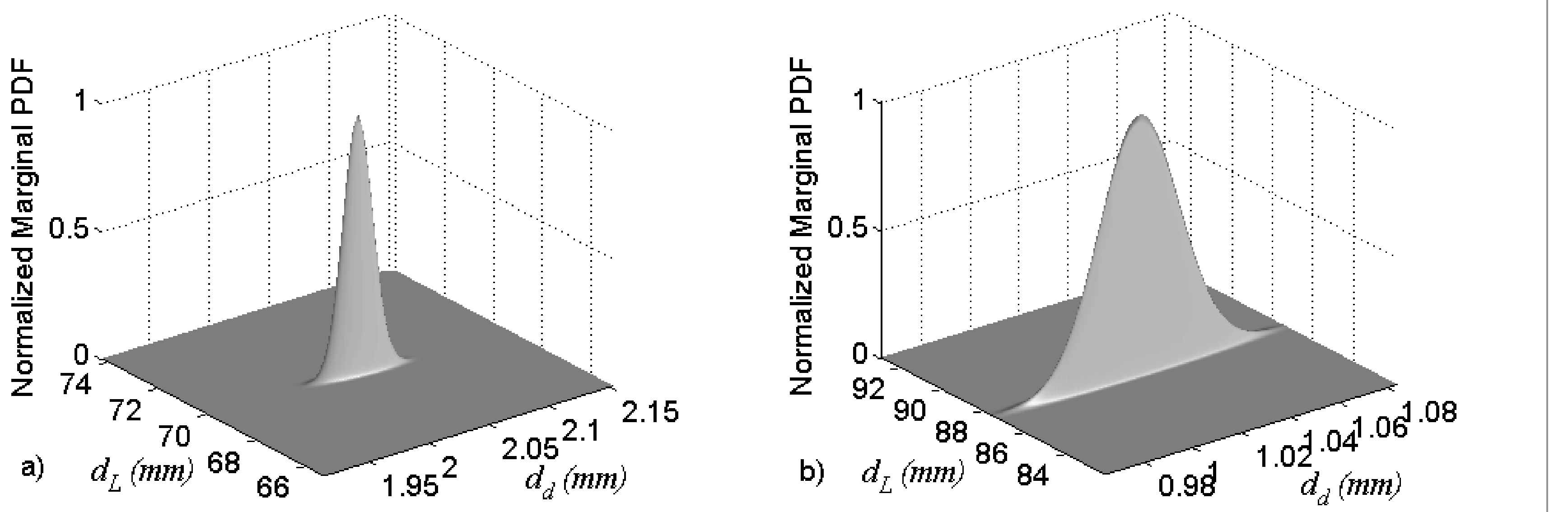
Case C2



Blue line: Experimental data

Red line: Spectral finite element simulation with identified damage parameters

Experimental Verification



Normalized marginal PDF of the identified damage length and depth for a) Case C1 and b) C2

Table 2. Summary of the results in damage identification

Case	Damage location (L_d) (mm)	Damage length (d_L) (mm)	Damage depth (d_d) (mm)
C1	1045.50 (0.0067%)	69.71 (0.1381%)	2.03 (0.5733%)
C2	902.88 (0.0123%)	87.46 (0.1267%)	1.02 (1.6179%)

Note: value in bracket is the coefficient of variation (COV)

Conclusions

- A method has been proposed to provide quantitative identification of damage in beams using longitudinal guided wave
- The method is able to identify damage location and size
- Frequency-domain spectral finite element has been employed to improve the computational efficiency
- The proposed method is also able to quantify the uncertainties associated with the damage identification results
- The proposed method has been experimentally verified
- The proposed method is currently extending to address the multiple damages situation and structures with complicated configurations

Thank You!